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AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

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Finding the External Force

THOMAS D. PHILLIPS
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(Received March 22, 1954)

Ambiguity in the statement of Newton's first law of motion occurs in most introductory physics texts. This is due to vagueness or inconsistency in the use of the word *body*. To make the statement of the law plain in meaning three revised wordings are available. In the opinion of the writer the best solution is to substitute the phrase, the *center of mass of any body* in the place of the common phrase *every body*.

IT may not be the exception that proves the rule, but it is the apparent exception that makes it clearer. For thousands of students the horse and cart problem has transformed the third law of motion from a vague assortment of words into an idea. After repeated explanations of this problem many teachers have begun to realize that part of the confusion arises from a vagueness in the usual statement of the law. Most textbook writers still use the traditional phrasing but include with it a modifying explanation which emphasizes that two bodies are involved. A few have courage to break with tradition and give a completely revised wording such as, "If body *A* exerts a force upon body *B* then body *B* exerts an equal and opposite force upon body *A*." The situation with regard to the third law of motion is reviewed here because it is generally recognized that confusion has arisen in the minds of students and that this confusion comes from lack of precision in stating the law.

A somewhat similar ambiguity which is present in the usual phrasing of Newton's first law of motion has received little attention. A few textbook writers have used their own revisions of this

law but in most first-year college books it still runs something like this: *Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed upon it.*

We teachers know, of course, what we mean by this law but unless we put some restrictions or special interpretations upon such a statement of the law our students would be justified in concluding that it is not always true. If our students are going to learn the law it ought to be so worded as to be clear, precise, and complete.

The looseness in statement is in the meaning of the word *body*. In most discussions this word remains undefined. The natural assumption for the student to make is that a body is anything that comes all in one piece. In the explanation of the horse and cart problem this idea is extended so that the horse and cart together are regarded as one body. It is quite in order, then, that the student should come to the conclusion that a *body* is anything you choose to draw a circle around.

If we accept this interpretation without reservations many exceptions to the first law of mo-

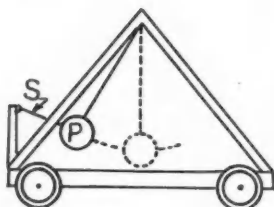


FIG. 1. The pendulum-cart. When the string S is burned the pendulum P swings and the cart moves.

tion come at once to mind. One such exception, the *pendulum-cart*, is shown in Fig. 1. It consists of a light cart supporting a heavy pendulum. The pendulum is held away from the perpendicular by a string which is tied to the frame of the cart. When the pendulum is released by burning the string the cart rolls back and forth along the table top and comes to rest in a new location.

If we regard the pendulum and cart as *one body* the only external forces acting are the force of gravity acting downward, the upward component of the force exerted by the table which is the equilibrant for the force of gravity, and the force of friction exerted by the table which acts in a direction opposite to that of the motion of the cart frame. There is no force external to the complete assembly to account for motion of the body. This experiment does not "disprove" the first law of motion but it does show that our usual statement of the law as quoted above is inadequate.

There are a number of possible wordings of the first law that make the law a true description of the behavior of the pendulum cart. These statements are as follows:

a. In place of the phrase *every body* we may say *any mass particle*. This takes care of the situation for a mass particle can have no internal motions. It is the internal motion of the pendulum that gives rise to the observed motion of the cart frame. The mass particle phrasing has the disadvantage that it is psychologically difficult for students. It has the still greater disadvantage that the law thus precisely stated for a mass particle is limited in value unless it may be applied to bodies having extension. To shift from mass particle to an extended body, such as the pendulum cart, is not always easy to do.

b. In place of the phrase *every body* we may say *every rigid body*. This also makes the statement

of the law an accurate description of events. If the pendulum cart were not permitted to have any internal motions, then the pendulum could not swing and the cart could not roll unless some other force were exerted by a body outside of the system. Limiting the application of the law to rigid bodies has, however, a serious defect. A rigid body is an idealized structure which has no counterpart in nature. This objection is of little consequence as there are many situations in which the rigid condition is sufficiently approached. The really serious objection is that this statement of the law eliminates a large group of phenomena and leaves us without a fundamental law to describe the motion of deformable bodies.

c. In place of the phrase *every body* we may say *the center of mass of any body*. So stated the first law avoids the need for admitting exception or for supplying an auxiliary interpretation. Instead of ruling the pendulum cart out of consideration, because it has internal motion and is nonrigid, it includes the device and makes its behavior easily predicted. Using this statement of the first law we can say in advance of experiment that if the pendulum is released and moves one way the frame of the cart will move the other way and the motion of the parts of the assembly will be such as to leave the center of mass at rest—neglecting friction. Further—since the force of friction cannot be completely eliminated—there will be *some* motion of the center of mass. Since the force of friction is in a direction opposite to that in which the frame of the cart rolls the actual motion of the center of mass of the assembly is in a direction opposite to that in which the cart frame first rolls.

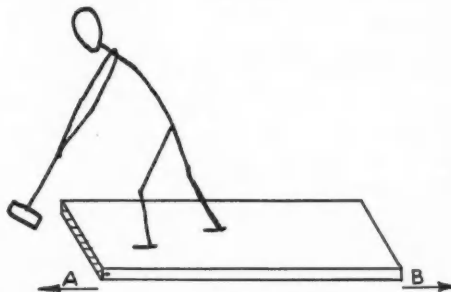


FIG. 2. The bump-mobile. The man propels the system of which he is a part by hitting the plank with the sledge.

Figures 2 and 3 illustrate two other devices which demonstrate the utility of rephrasing the first law of motion so as to focus attention on the behavior of the center of mass.

Figure 2 is a *bump-mobile*. It consists of a wooden plank, a heavy sledge hammer and a man. The man stands on the plank and hits the end of it with the sledge. If we accept the definition that the plank, sledge, and man are *one body* then we find that though the plank remains at rest the center of mass of the assembly moves in the direction of arrow *A* as the man raises his sledge to strike and the force of "static" friction acts in this direction to "cause" the motion. As the man swings the sledge toward the end of the plank the center of mass of the system acquires a momentum in the direction of *B* and the force of "static" friction acts in that direction. When the sledge hits the plank the internal motion stops but the motion of the center of the system continues for a bit with the force of sliding friction of the plank moving along the floor opposing this motion.

Figure 3 is a *sway-mobile*. It consists of a platform cart with a follower-chock. The chock leaves the cart free to move in one direction but prevents the wheels from rolling in the other direction. The operator stands on the platform and by swaying his body propels the machine with considerable speed along a level floor. The chock is arranged so the cart cannot roll in the direction of *A*. If we draw a circle around the man and the cart and say the two together are to be regarded as one body then when the man sways in the direction of *B* the cart part remains at rest but the center of mass of the system moves in the direction of *B* and the force of "static" friction supplied by the chock acts in the direction of *B*. When now the man sways in the direction of *A*, the cart rolls in the direction of *B*. Except for the retarding effect of force of friction

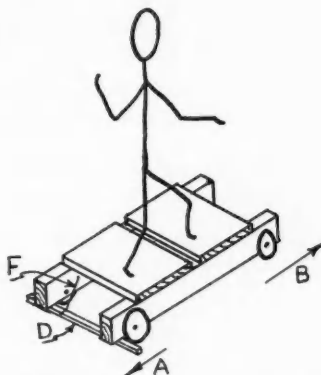


FIG. 3. The sway-mobile. The platform has its wheels blocked from rolling in one direction by the stick *D* which is fastened to the platform by strings *F*. The man propels the cart by swaying.

acting in the direction of *A*, this motion of the cart will be such as to continue the center of mass at rest.

Using the phrase "the center of mass of any body" in the first law of motion has one disadvantage. With some writers it is the custom to use Newton's second law of motion as the basis for a definition of mass. If this is done, then the use of the notion of center of mass in the statement of the first law is a bit premature. This objection can be eliminated by defining mass prior to making any statement of the laws of motion at all. This is not hard to do nor do we need to give up the idea of inertial mass in order to do it.

Students often learn the laws as printed in the textbooks. It seems only reasonable to ask that the statements of these laws be true as they stand without the need of too much specialized interpretation and extended discussion of situations to which they do not apply. It is suggested that in introductory physics texts the wording of the first law of motion be changed to read: *the center of mass of any body continues, etc.*

The most difficult thing for a teacher is to recollect how much it cost him to learn, and to accommodate his instruction to the apprehension of the uninformed.—Thomas Young, *Natural Philosopher* by ALEX WOOD (Cambridge University Press, 1954).

Topology of Steady Current Magnetic Fields

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(Received December 14, 1953)

Textbooks often hold erroneously that the condition $\nabla \cdot \mathbf{H} = 0$ implies that lines and tubes of force are closed. Examples are given in which the lines of force are not only unclosed, but also, the density of lines may be made infinite in a region where the field intensity is finite. If the steady electric current density is everywhere finite, the magnetic field has singularities only at discrete points, lines, or surfaces, where $H = 0$. In general, an infinity of lines of force may terminate on these points. Lines of force may have finite length, going from one singularity to another. A systematic analysis is made of the allowed distribution of singular points, and the linear fields in the neighborhoods of these points are classified. Certain types of singularities may appear in regions of finite current density, but cannot occur in regions free of current. Lines of force may form limit cycles.

I. INTRODUCTION

SINCE their invention by Faraday, assemblages of lines of force have been used extensively in the analysis and description of static magnetic fields. We are accustomed to imagine these assemblages drawn such that their cross-sectional density is everywhere proportional to the magnetic intensity, and we have regarded the energy density as being distributed throughout the field proportional to the square of the number of lines per unit cross-sectional area. Although the efficacy of these concepts is unquestioned, there is ample room here for criticism since, as Slepian¹ pointed out, there frequently arise misconceptions about these assemblages. We know of no estimable investigation of the topological properties of static magnetic fields. Perhaps this is the reason for these common fallacies. The aim of this paper is to reveal these misconceptions, common amongst authors of both elementary and advanced textbooks, and to catalog some of the important topological properties of magnetic fields.

To arrive at the root of these fallacies, let us briefly review the mathematical relations expressing the properties of assemblages of lines. A magnetic *line of force* is defined as a curve which is everywhere tangent to the field intensity \mathbf{H} . If s_i is a component arc length parameter in generalized coordinates $u^i(x, y, z)$, where $ds_i = h_i du^i$ ($i = 1, 2, 3$), the differential equations of

the lines of force are

$$\frac{h_1 du^1}{H_1} = \frac{h_2 du^2}{H_2} = \frac{h_3 du^3}{H_3}.$$

The solution of these equations, in general, contains two parameters which serve to individualize a line of force. The equations are valid for any material medium, both within and without the current regions. External to the current regions, the lines form the orthogonal trajectories of the magnetic potential surfaces.

To deal with assemblages of these lines, let us now investigate the implications of the divergenceless character of \mathbf{H} .² The divergence theorem requires the equality of the flux through the two ends of a tube of force, that is, a tube whose bounding surface is made up of lines of force. From this fact, it is commonly inferred that lines of force are always uniquely defined, continuous, never-ending, curves which either close on themselves or pass off to infinity. If the line density in an assemblage is chosen proportional to H in an initial region, the conclusion is that the lines may *always* be prolonged away from this region and the assemblage will continue, by its density, to represent H , the lines spreading where the field diminishes and converging where it strengthens. There are two principal reasons why these conclusions are false. In the first place, there usually exist singular points in a magnetic field in which $H = 0$, and

² In what follows, we consider static magnetic fields generated by macroscopic current distributions. For simplicity, we assume a uniform permeability equal to unity.

¹ Joseph Slepian, Am. J. Phys. 19, 87 (1951).

where the direction of the magnetic vector is not uniquely specified. If a tube of flux initiates in a regular region and is prolonged so as to include either within it or on its surface one of these singular points, the tube may branch out into several tubes and there may be an infinity of lines which end on these points. Lines of force may, in fact, have finite length. Secondly, lines of force are generally not closed curves (similarly in the steady state flow of electric charge, the condition $\nabla \cdot \mathbf{J} = 0$ in no way implies that the current filaments are closed). For this reason, as we shall see in Part II, unless artificial space cuts are employed, it is possible to construct fields in which the line density is infinite in a region where the magnetic intensity is finite. Such regions could be dealt with in a straightforward way if there were general criteria which decided whether the lines form closed curves. Unfortunately, such criteria are unknown.

In Part II we shall point out some of the difficulties encountered in attempting to establish these criteria. In Part III we define the types of singularities which arise in static magnetic fields and in Part IV we describe the types of linear fields which may occur in the neighborhoods of these singular points. Part V is devoted to the development of a theorem on the allowed distribution of singular points. In Part VI we shall discuss briefly the application of the Poincaré index to plane two-dimensional fields, and in Part VII we investigate those limiting closed space curves (limit cycles) which are approached asymptotically by the lines of force.

Finally, we should comment on the one obvious, objectionable feature of representing a vector field by assemblages of individual lines. Everyone has experienced the unpleasant intuitive feeling concerning those regions where H is relatively weak: If the field is represented by ten lines per square centimeter at a point P where the magnitude of \mathbf{H} is $H(P)$, how do we represent the field at point P' where its magnitude is $H(P') = \pi^{-1}H(P)$? Some authors have attempted to circumvent this difficulty by imagining tubes of force constructed from an equal, but arbitrarily large, number of lines of force, the number N of tubes per unit cross-sectional area being equal to the local value of H . It is then held that through any element of

area δS

$$\delta N = \mathbf{H} \cdot \delta \mathbf{S}.$$

This is an obvious attempt to continuously distribute the lines so as to cover all points of space. The replacement of the concept of line density by that of tube density but little improves our position.

II. ON THE CLOSING OF LINES OF FORCE

Magnetic lines of force and of induction are not generally closed curves. To establish our argument let us examine the field generated by a fixed current I_1 flowing in a ring solenoid and an adjustable current I_2 flowing along its axis, Fig. 1. A line of force, produced by the ring solenoid alone ($I_2 = 0$), which originates at a point P will link the circuit of I_1 and return to P , the line always remaining in the plane through the axis and point P . Now if we insert the circuit I_2 , we see, from the right-hand rule, that the field of I_2 produces the following effects, depending on the magnitude of I_2 : The lines of force originating at P will link both I_1 and I_2 and (a) return to P after an integral number of linkages n of I_1 and m of I_2 , or (b) will never return to P (incommensurable case). Thus, if we start an assemblage of lines from a two-dimensional region R and follow it continuously around the wire, we find that the tube of this assemblage does not return to the individual points where it originated. Moreover, similarly constructed adjacent tubes will spiral about the solenoid and eventually enter the region R , their lines becoming interlaced with lines originating in R . In the incommensurable case we should therefore find an infinite line density although H is finite. If we wish to retain the concept of line density as representing H , then the lines must not be continued on into the initial region. The lines must be interrupted by introducing artificial surface cuts. The lines which are stopped in this way must have a cross-sectional line density equal to the density of lines which originate on the opposite side of the cut, to preserve the condition $\nabla \cdot \mathbf{H} = 0$. Thus we may construct a satisfactory representation of the field using assemblages of lines even when the lines do not close. In our example, the lines must be artificially broken at least once for each revolution about the wire if they are to have the proper density.

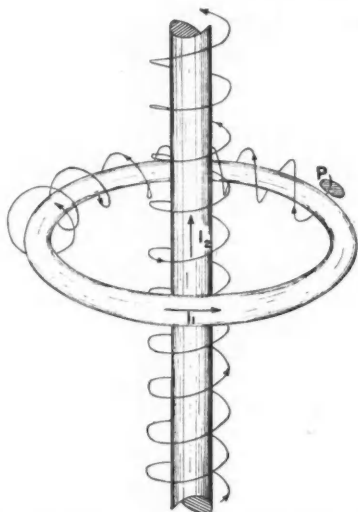


FIG. 1. Nonclosed lines of force produced by the steady currents I_1, I_2 .

General criteria which decide whether the lines close on themselves have not been established. It can only be said that definite symmetry of some sort must exist in the case of closed lines. On the other hand, systems of *conductors* exhibiting definite symmetry do not always form closed lines of force. Any plane through the axis of the straight conductor of Fig. 1 is such a symmetry plane of the conductors. A second, trivial example is that of a straight wire conductor parallel to the Earth's constant magnetic field; the lines form helices.

To illustrate the difficulties encountered in attempting to establish these criteria, we consider one example. In Fig. 2a is shown an arbitrary current filament i_1 which is reflected in the XY plane to obtain the current filament i_2 . To compute the field intensity at an arbitrary point P we employ the law of Biot-Savart, performing the integration on both circuits simultaneously (in the figure we have $\xi_1 = \xi_2$, $\eta_1 = \eta_2$, $\zeta_1 = -\zeta_2$). The analysis is sufficiently straightforward to allow us to state only the results. There are two cases of interest. If i_2 has the direction shown in the figure, i.e., $i_1 = i_2$, we find that H_x and H_y are odd functions of z , whereas H_z is even. This implies that the field below the XY plane is the reflection of that above, except that the sense of

the lines of force is reversed. This is a sufficient condition for a line of force to close on itself provided it penetrates the XY plane twice. However, it may penetrate the XY plane only once and thereafter either go to infinity or end in a singular point in the finite domain. Also, it may not intersect the XY plane even once. In the latter case, a line may spiral about the circuit, connect different singular points, or connect a singular point with infinity. In the second case ($i_2 = -i_1$) we find that H_x and H_y are both even functions of z , whereas H_z is odd. Here we have a true reflection of the magnetic field in the XY plane, including the sense of the lines. Evidently, we can draw no important conclusions from this case.

It is commonly held that plane circuits of otherwise arbitrary shape always produce closed lines of force. Our above analysis immediately shows the falsity of this statement. For we may set $\zeta_1 = \zeta_2 = 0$ in the first case above ($i_1 = i_2$). Then both circuits coincide and lie in the XY plane. Again we conclude that if a line penetrates the XY plane twice, it necessarily closes on itself. However, it may penetrate this plane once and subsequently go to infinity or end on a singular point, or it may not intersect the plane even once. A very trivial example is shown in Fig. 2b. The single loop of wire is deformed so as to approximate two similar, closed, circular loops. A map of the lines on the equidistant plane resembles that due to a pair of oppositely charged discrete poles.

III. DEFINITIONS OF SINGULAR POINTS

A steady current magnetic field contains only two types of singularities at a point or set of points $\{P_0\}$:

(a) The magnitude of \mathbf{H} is infinite at $P_0(x, y, z)$. This type of singularity is artificially synthesized and cannot occur in macroscopic current distributions of finite current densities. We have no interest here in this singularity.

(b) The direction of \mathbf{H} is not uniquely defined at P_0 . The existence of this type of singularity requires that H vanish at P_0 . An immediate example is offered by the field of an infinite solid cylindrical conductor of radius a and uniform current density \mathbf{J} . Near the axis of the wire

the lines of force are concentric circles and the only nonvanishing component of the field intensity is the angular component $H_\phi = 2\pi J r / c$ (Gaussian units). The point P_0 on the axis contains a circular line of force of radius zero. Also $H(P_0) = 0$. Thus we conclude that the axis of the wire is a line distribution of singularities—a line singularity.

We consider one further example. Suppose we remove from the above cylinder a solid cylindrical section of radius $b < a$, Fig. 3. In the region $b \leq r \leq a$ the only nonvanishing component of the field intensity is the angular component $H_\phi = (2\pi J / cr)(r^2 - b^2)$. Consider the field in the neighborhood of a point P_0 on the surface $r = b$. For values of $r \leq b$, $H = 0$ so that \mathbf{H} may be assigned any direction. For values of $r > b$, the field is essentially tangent to the cylindrical surface at P_0 . The surface $r = b$ is a surface distribution of singular points, since the direction of \mathbf{H} is not uniquely defined in the neighborhood of any point P_0 of this surface. Finally, when H vanishes identically at points in a volume distribution, it is true that we may assign any direction to \mathbf{H} . However, we choose not to classify such points as singular points. Thus, in Fig. 3, the points for which $r < b$ are not counted as singular points.

To investigate the behavior of the field in the neighborhood of a singular point, we next classify the field according to the coefficients which appear in the linear terms of its Taylor expansion.

IV. LINEAR FIELDS

We consider the first-order terms in the Taylor expansion of an arbitrary vector field about

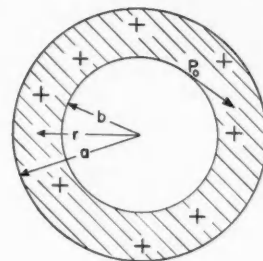


FIG. 3. Surface distribution of singular points.

some point P_0 for which the components of the vector are regular. Such a *linear field* may always be represented as the sum of a vector translation field, vector rotation field, and a homogeneous deformation field.³ Thus we may write, for an arbitrary vector \mathbf{F} ,

$$\mathbf{F}(x, y, z) = \mathbf{F}_0 + \frac{1}{2}(\nabla \times \mathbf{F})_0 \times \mathbf{R} + \hat{i}_1(a+b)x + \hat{i}_2(a-b)y + \hat{i}_3cz, \quad (1)$$

where a, b, c are real constants and $\mathbf{R} = \hat{i}_1x + \hat{i}_2y + \hat{i}_3z$. The rectangular axes are here chosen so that they are the principal axes of the deformation field. For our specific purpose, we equate \mathbf{F} and \mathbf{H} , and we regard P_0 as a singular point of the field of \mathbf{H} . Thus the vector translation field vanishes. If we write Eq. (1) in matrix form, we find that the rotation vector contributes three antisymmetric elements to the coefficient matrix, whereas the deformation vector contributes the diagonal elements. Moreover, the divergenceless character of \mathbf{H} requires that the trace of the matrix vanish. Hence we are left with five independent parameters and our matrix equation may be written

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} a+b & -A & -B \\ A & a-b & -C \\ B & C & -2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad (2)$$

According to the five parameters, we specify five independent component fields, Fig. 4.⁴ The

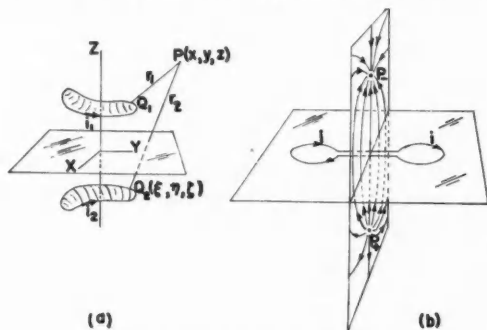


FIG. 2. (a) Reflected circuits. (b) Field of a plane current filament.

³ See H. B. Phillips, *Vector Analysis* (John Wiley and Sons, Inc., New York, 1948), p. 44.

⁴ Our choice of resolving an arbitrary linear field into its five component fields is not unique. For example, a field may consist of three deformation fields, one lying in each coordinate plane. Our analysis merely shows that principal axes may be so chosen that the three deformation fields are equivalent to some combination of the five fields shown in Fig. 4. Also, there may arise singularities in which several different types of fields occur about the same singular point, depending on the solid angular sector about this point. Figure 6c is such an example.

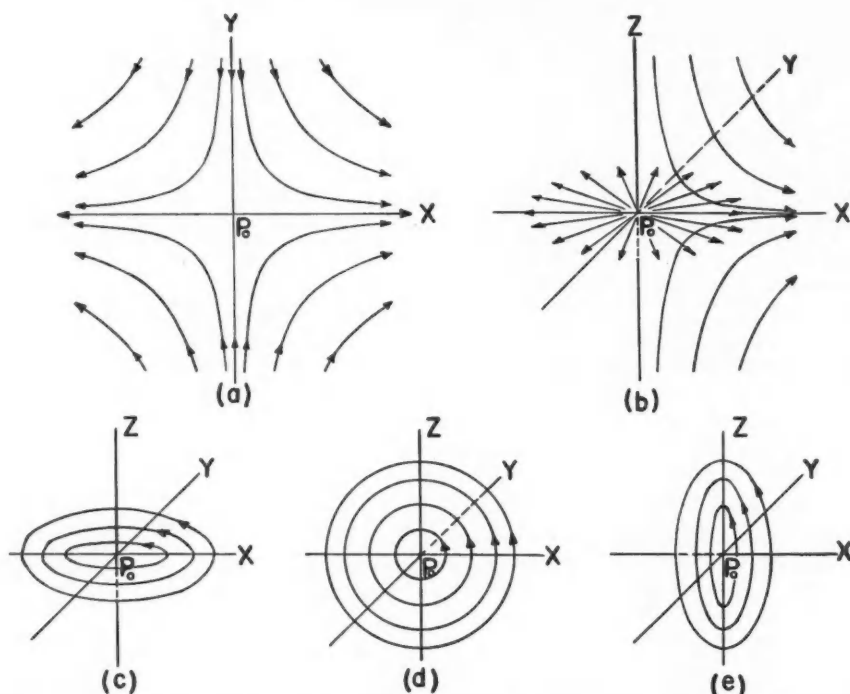


FIG. 4. Component linear fields in the neighborhood of a singular point. (a) Deformation field ($b > 0$), $xy = \text{constant}$. (b) Quasi-divergence field ($a > 0$), $x = k_1 y$, $zr^2 = k_2$, $r^2 = x^2 + y^2$. (c) Rotation field ($A > 0$), $x^2 + y^2 = \text{constant}$. (d) Rotation field ($B > 0$), $x^2 + z^2 = \text{constant}$. (e) Rotation field ($C > 0$), $y^2 + z^2 = \text{constant}$.

three rotation fields appear as pure rotation fields in each of the three coordinate planes. The deformation field is, in our representation, a plane two-dimensional field consisting of the family of equilateral hyperbolas $xy = \text{constant}$. (Observe that the lines in the plane $x=0$ intersect P_0 with opposite senses—similarly for the lines in the plane $y=0$. We may therefore choose to say that these lines terminate and begin at P_0 , respectively. Such a two-dimensional field would therefore contain an infinity of lines terminating and initiating on the Z axis.) The quasi-divergence field has rotational symmetry about the Z axis. In the plane $z=0$, the field is a two-dimensional divergence field with the lines of force extending radially away from P_0 (for $a > 0$). Since the lines all diverge (or converge) at P_0 , they must terminate there; otherwise, they do not retain their uniqueness property. Yet the three-dimensional field is divergenceless. Indeed, it is possible to have several two-

dimensional nondivergenceless fields intersect at a point and still preserve a three-dimensional divergenceless condition.⁵ This apparent paradox is easily resolved. Let us pass a regular surface S' through a simply connected region R bounded by a regular surface S . S' intersects S is a closed curve C . We conclude that the vanishing of the flux integral over S imposes no immediate restriction on the behavior of the field over S' , since S' has no thickness and thus contributes nothing to the surface integral for that portion of the integration near curve C .

It is of interest to observe the manner in which a tube of force would be deformed if it originated in a region which included the Z axis of Fig. 4b; the tube would fan out and cover the whole XY plane. Also observe the resultant deforma-

⁵ By a "two-dimensional field" we mean any regular surface over which the normal component H_n of the field vanishes. This imposes no restriction on the normal derivative of this component.

tion of a tube of force originating in a region which contains one of the axes in Fig. 4a; a single tube splits into two tubes.

An example of a quasi-convergence field is shown in Fig. 5. The field due to the currents I_2 in the circular conductors produce a quasi-convergence field in the neighborhood of P_0 . If we now allow a current I_1 to flow in the solid axial conductor, we have a pure rotation field superposed on our previous field, thereby causing the lines in the XY plane to spiral in toward P_0 .

We now examine plane two-dimensional fields. Without loss of generality we set $H_z \equiv 0$ in Eq. (2), so that in the XY plane, $B = C = 0$. Thus, for a plane two-dimensional linear magnetic field, valid in the neighborhood of a singular point P_0 , we have

$$\begin{bmatrix} H_x \\ H_y \end{bmatrix} = \begin{bmatrix} a+b & -A \\ A & a-b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad (3)$$

which is resolved into the rotation field (Fig. 4c), the deformation field (Fig. 4a) and the divergence field lying in the XY plane of Fig. 4b. If the two-dimensional field is divergenceless, only the rotation and deformation fields are allowed. These are both represented in the following example, Fig. 6: This plane two-dimensional divergenceless field is produced by an infinite straight conductor of uniform current density, immersed in a uniform field of strength H_{0z} . The lines of force within the current region are the family of circles $x^2 + (y - cH_{0z}/2\pi J)^2 = k^2$, of parameter k . Exterior to current region the lines are represented by $r = k_1 \exp\{ycH_{0z}/2\pi a^2 J\}$, where $r^2 = x^2 + y^2$ and a is the radius of the wire. The occurrence of singular points depends on the

relative strengths of H_{0z} and J . In Fig. 6b there occurs a rotation field at $P_0(0, cH_{0z}/2\pi J)$, and a deformation field at $P_1(0, 2\pi a^2 J/cH_{0z})$ with principal axes rotated through 45° . This case is characterized by the relation $cH_{0z}/2\pi J < a$. In Fig. 6c, $cH_{0z}/2\pi J = a$ and the two singular points fuse into one at $P_0(0, a)$, the sector of 180° on the left comprising a deformation field and that on the right a rotation field. In Fig. 6d, $cH_{0z}/2\pi J > a$ and there are no singular points. The dashed lines of force have been sketched in only to indicate the general nature of the field without attempting to preserve the proportionality between line density and field strength.

In this chapter we have discussed only the structurally simplest types of fields possible. It may happen that the field in the neighborhood of a singular point cannot be represented as a linear field. The field of three parallel cylindrical conductors of infinite extent, placed at the vertices of an equilateral triangle, contains a singular point at the center of the triangle when the currents have the same sense. But the lowest terms in the Taylor expansion about this point are of second order. We therefore have a singular point of *second order*. Singularities of second and higher order would require a separate investigation.

V. DISTRIBUTION OF SINGULARITIES

Certain types of singularities may appear in regions of finite current density, but cannot occur in regions free of current. In this chapter, we wish to catalog the allowed types of singularities in these two regions.

Our previous examples have established that discrete point and line singularities may exist

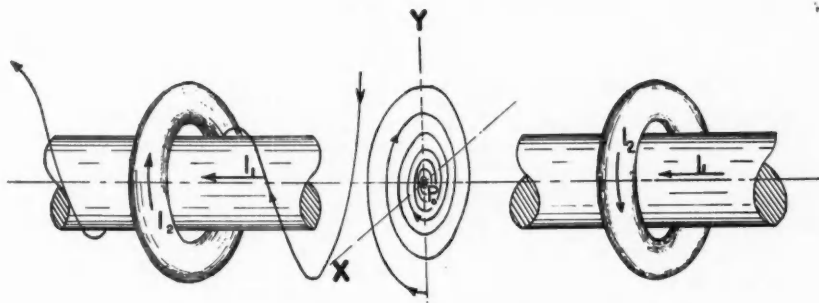


FIG. 5. Superposition of a quasi-divergence field and a rotation field.

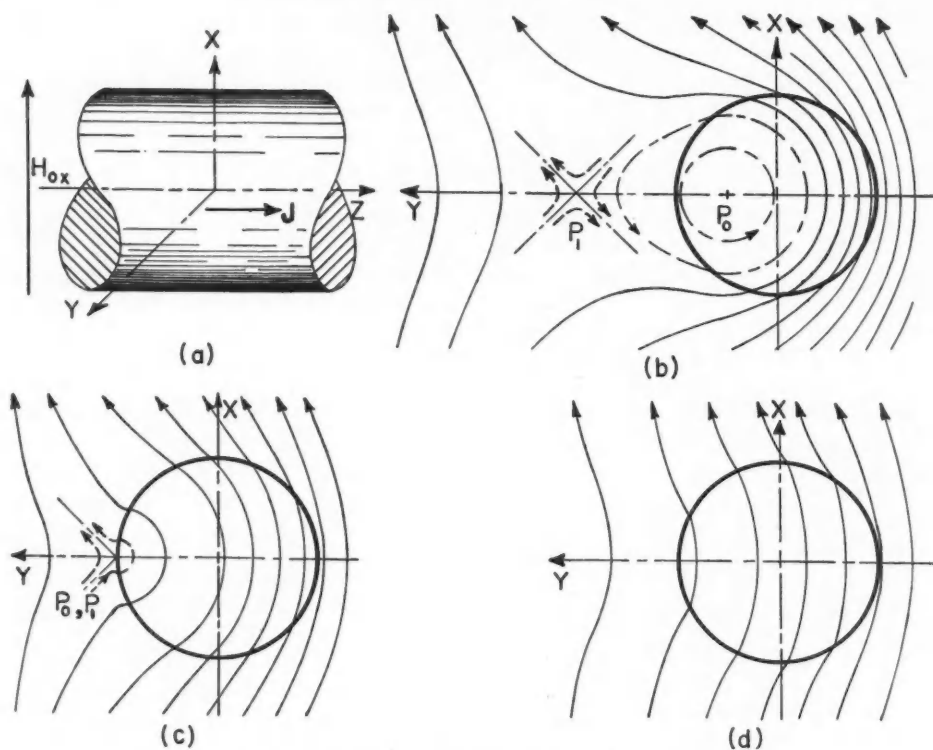


FIG. 6. Singular points of plane two-dimensional divergenceless field.

both within and without the current regions. A distinct line singularity may pierce the boundary of the current region and thereby be contained in both of these regions. In general, a magnetic field will contain curved line singularities, several of which may intersect in a common point. A line singularity may form a closed curve. In this case, the neighboring lines of force may or may not be asymptotic to the curve. If the lines are asymptotic, we say that the curve forms a limit cycle (the latter is discussed in Chap. VII).

Of the possible distributions of first-order singularities external to current regions, only the deformation and quasi-divergence types are allowed, the component rotation fields being ruled out by Ampère's law. Since line singularities are surrounded by two-dimensional fields, it follows that these fields are necessarily of the pure deformation type.

We have yet to catalog the occurrence of surface distributions of singular points. In Fig. 3,

we observed that the inner surface of the cylindrical shell comprised a surface of singularities. Now suppose that we surround this shell with a second cylindrical shell, concentrically. By adjusting the sense and magnitude of the current in the outer shell, we are easily convinced that a second cylindrical surface of singularities may be generated which always lies within or on the boundary of the current region. Adding more shells in no way enables us to generate a surface distribution of singular points which lies external to the current region.

A second trivial example is that of the plane infinite current sheet of thickness h and uniform current density $J=J_z$, shown in Fig. 7. Using Ampère's law we find that

$$H_z(y) = \left[1 - \frac{2y}{h} \right] H_0, \quad H_0 = \frac{2\pi h}{c} J, \quad H_y = H_x = 0,$$

where H_0 denotes the constant field external to

the current region. The field vanishes over the surface of singularities at $y=h/2$. This plane two-dimensional divergenceless field is linear and may therefore be resolved into a rotation field and a deformation field. Again the singular points occur within the current region. To prove that surface distributions of singular points always occur within the current regions ($J \neq 0$), or on their boundaries, let us assume the contrary and thereby prove a contradiction.

Let σ denote a regular surface of singular points lying external to the current region. For our purpose we insert the necessary space cuts which ensure the vanishing of the magnetomotive force for every allowed closed path. This is a n.a.s.c. that \mathbf{H} be the gradient of a scalar ϕ , valid in a domain τ not containing the space cuts. It follows that ϕ is a harmonic function in τ since the divergence of \mathbf{H} is identically zero. The vanishing of H over σ implies that, for points of σ , $\phi = \text{constant}$ and the normal derivative of ϕ vanishes. But if a function ϕ is harmonic in a domain τ and is constant at all points of any finite surface element, however small, contained within or on the boundary of τ , and further, if the normal derivative of ϕ vanishes over this element, then ϕ has this constant value at all points of τ .⁶ This implies $H \equiv 0$ in τ —a contradiction of our original hypothesis. We thus have proved that surface distributions of singular points always occur within or on the boundaries of current regions. From our previous examples we may now state the *theorem*: *Discrete point, line, and surface singularities generated by steady current magnetic fields may be found either within or without the current regions, with the single exception that surface singularities are confined to the current regions or their boundaries.*

From the method of proof it is also evident that if H vanishes identically over some finite portion of the boundary of a current region, it vanishes identically over all the connected boundary, and over all the connected space external to the current region.

It is worthwhile to apply this theorem to two-dimensional fields generated by an arbitrary distribution of infinite parallel current filaments. In the plane perpendicular to the current filaments, we have the *Corollary*: Discrete point and line singularities generated in plane two-dimensional fields by *infinite parallel current filaments*, may be found either within or without the current regions, with the exception that line distributions of singular points always occur within or on the boundaries of current regions. Figures 3 and 7 verify the corollary. The corollary may be proved directly by use of Green's theorem for the plane.

VI. THE POINCARÉ INDEX

In a plane two-dimensional field we may choose in the XY plane a simple closed curve Γ which does not cross singular points. If we move a point P along Γ , the field vector $\mathbf{h} = \mathbf{H}/H$ will rotate so that after one complete circuit of P on Γ an integral number j of revolutions of \mathbf{h} will be experienced. We shall choose j positive when \mathbf{h} rotates counterclockwise, in accord with the usual direction of integration. We call j the index of the closed curve Γ with respect to the field of \mathbf{H} . Evidently Γ may be deformed continuously without changing the value of the integral (since j is an integer), so long as no singular points are intercepted. Thus we may shrink the curve Γ about a discrete singular joint and thereby calculate its index. These concepts are formu-

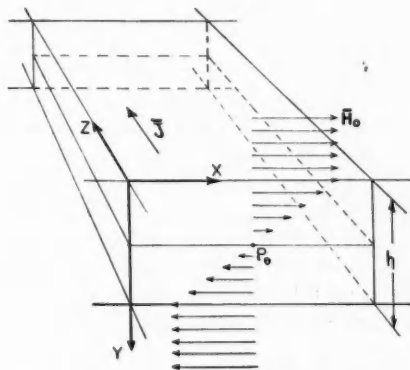


FIG. 7. Surface distribution of singular points generated by infinite plane current sheet.

⁶ See F. Pockels, *Über die partielle Differentialgleichung $\Delta u + k^2 u = 0$* (Teubner, Leipzig, 1891), pp. 212–213. The theorem, applied to the wave equation, is also valid for Laplace's equation ($k=0$).

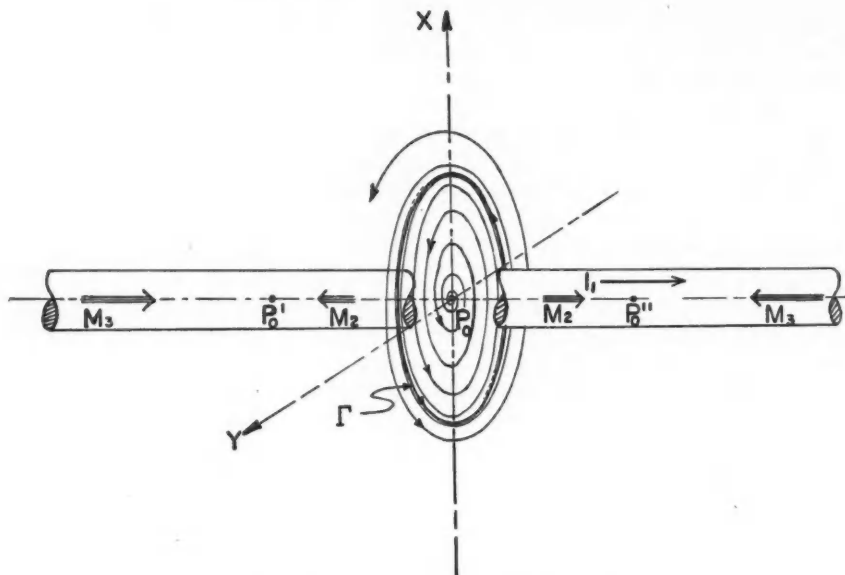


FIG. 8. Limit cycle external to current region.

lated below:

$$j = \frac{1}{2\pi} \oint_{\Gamma} d\theta = \frac{1}{2\pi} \oint_{\Gamma} d\left(\tan^{-1} \frac{H_y}{H_x}\right) \\ = \frac{1}{2\pi} \oint_{\Gamma} \frac{H_x dH_y - H_y dH_x}{H_x^2 + H_y^2}. \quad (4)$$

Since this a curvilinear integral of a total differential, the continuity of the function over the surface bounded by Γ requires that j vanish. That is, the index of a closed curve which does not surround singular points is zero. Unfortunately, the converse of this is not true, as one may verify from Fig. 6c. A closed line of force surrounds at least one singular point. For the index of this curve is always $+1$ regardless of the sense of the line of force. Equation (4) may be extended to general two-dimensional fields.

It is of interest to compute the index for the plane two-dimensional field given by Eq. (3). By choosing as path of integration the ellipse Γ , $[(a+b)x - yA]^2 + [xA + (a-b)y]^2 = 1$, we have, after a simple calculation,

$$j = \frac{a^2 - b^2 + A^2}{2\pi} \oint_{\Gamma} (xdy - ydx).$$

But $\frac{1}{2} \oint_{\Gamma} (xdy - ydx)$ is the area $\pi/|a^2 - b^2 + A^2|$ of the ellipse. Thus we have

$$j = \frac{a^2 - b^2 + A^2}{|a^2 - b^2 + A^2|} = \pm 1.$$

The singular case $a^2 - b^2 + A^2 = 0$ does not pertain to discrete singular points. Rather, we deal here with a line of singularities which cannot be surrounded by Γ . We conclude that a pure deformation field has a negative index, regardless of the sign of b . Similarly, both the divergence (or convergence) field and the rotation field have positive indices, independent of the signs of a and A .

It should be pointed out that the theory of plane two-dimensional vector fields has been thoroughly investigated in connection with the theory of oscillations. To illustrate this we observe that if t is an arc length parameter, we are justified in writing the two-dimensional equations of the lines of force in the form

$$\frac{dx}{dt} = H_x(x, y), \quad \frac{dy}{dt} = H_y(x, y), \quad (5)$$

since the slopes of the lines are correctly given by $dy/dx = H_y/H_x$. These equations have been

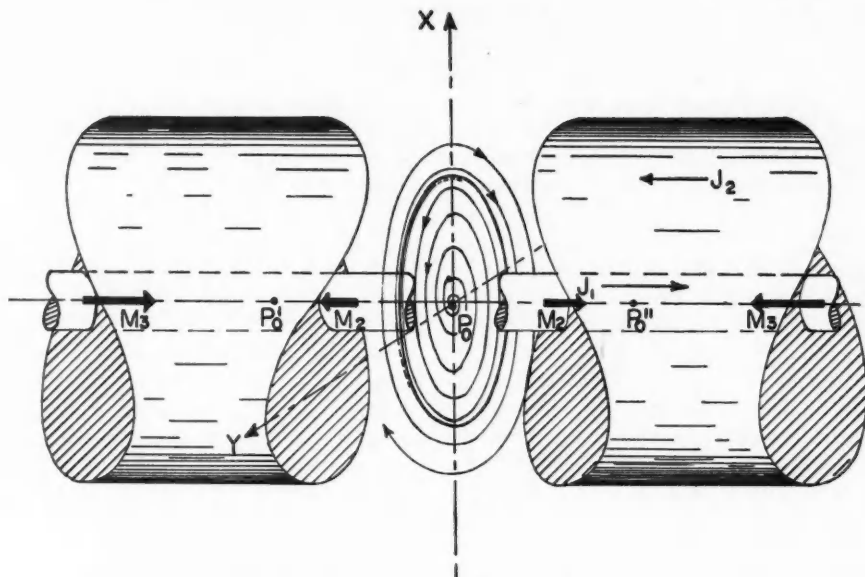


FIG. 9. Limit cycle composed of singular points.

thoroughly investigated in connection with autonomous dynamical systems wherein the parameter t is measured on a time axis, and for the most part we thus merely need a change of nomenclature.⁷

VII. LIMITING CURVES (LIMIT CYCLES)

We now investigate limiting closed space curves which are approached asymptotically by the lines of force. We shall hereafter refer to all such curves as *limit cycles* in agreement with Poincaré's terminology for isolated periodic motions. This terminology has advantages since, as we have remarked, the theory of oscillations is directly applicable here for the case of plane two-dimensional fields.

The existence of a limit cycle Γ requires that the latter surround current elements, unless all its points are singular points, except for a set of measure zero. For from Ampère's law we have

$$\oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = \frac{4\pi I}{c},$$

⁷ See, for example, *Theory of Oscillations* by A. A. Andronow and C. E. Chaikin (Princeton University Press, Princeton, 1949), Chapters 5, 6, 7.

which proves our statement since the magnetomotive force around such a limit cycle cannot vanish. Limit cycles, however, are not restricted to current regions. This point is illustrated in the following interesting example: To the circuits shown in Fig. 5, let us add two further ring solenoids, each with currents I_3 , about the straight wire conductor, their planes lying parallel to the XY plane and located symmetrically with respect to the latter at a distance greater than that of the two previous solenoids. This arrangement is illustrated in Fig. 8, wherein the ring solenoids are simply represented by the heavy arrows of appropriate directions. The field of the ring solenoids in the XY plane is directed radially toward or away from P_0 . By making I_3 considerably larger than I_2 we may locate a circle Γ on which the radial field vanishes, the field outside the circle being divergent away from P_0 and the field inside being convergent toward P_0 . By closing the circuit of I_1 in the straight wire solenoid, we thus produce a limit cycle, the circle Γ , lying external to the current region. Obviously we may continue this process and obtain any number of limit cycles lying in the XY plane. It is interesting to observe that our process has also generated two further singular

points P_0' , P_0'' of the same type as P_0 . The axial lines of force connecting P_0 , P_0' and P_0 , P_0'' are of finite length (similarly, the lines connecting P_+ and P_- in Fig. 2b have finite lengths).

We may also construct limit cycles which consist entirely of singular points. Such an example is illustrated in Fig. 9. The arrangement is similar to that of Fig. 8 wherein the previous ring solenoids are again represented by heavy arrows. We recall that their field in the XY plane is radially converging toward P_0 in the region inside Γ and radially diverging in the region outside, vanishing at all points of Γ . We wish to apply a rotation field which vanishes on Γ . Instead of the single straight wire conductor we employ two coaxial straight wire conductors with

oppositely directed currents, adjusted so that the angular field intensity vanishes at points of Γ . This gives our desired construction. It should be observed that points of Γ lie either within or on the boundary of the current region of the outer conductor, such that the net current passing through Γ is zero.

The existence of limit cycles, consisting entirely of singular points which lie external to current regions is impossible; otherwise, we would have a violation of Ampère's law.

Limit cycles of the types shown (i.e., where the asymptotic lines lie in a plane surface), cannot exist if $\partial H_z/\partial z$ has the same sign at all points of the region under investigation. For from Green's theorem for the plane, the integral

$$-\iint_S \frac{\partial H_z}{\partial z} dx dy = \iint_S \left(\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} \right) dx dy = \oint_\Gamma (H_x dy - H_y dx) \quad (6)$$

cannot vanish since the integrand of the surface integral does not change sign. But on Γ we know that $dx/H_x = dy/H_y$ and the line integral must therefore vanish. This contradiction proves the statement. Equation (6) may likewise be useful in deciding whether a line of force in the XY plane closes on itself. Frequently a casual inspection of the surface integral is sufficient, even when the integrand changes sign.

In a two-dimensional divergenceless field produced by infinite parallel current filaments of arbitrary distribution, limit cycles can never occur. Suppose such a limit cycle to exist. We can describe a rectangular prism whose bases are perpendicular to the current filaments and whose contours lie just inside the limit cycle. The three-

dimensional divergenceless condition then requires that the flux of \mathbf{H} vanish over this prism. Since the flux is zero through the individual bases it must vanish over the lateral surface, but this is impossible since the lines of force are asymptotic to the limit cycle. We thus have a contradiction.

In this chapter we have only pointed out some of the elementary and obvious properties of limit cycles. Actually, little is known on this subject even in the theory of oscillations.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Professor W. M. Elsasser for numerous helpful suggestions and valuable criticism.

Spectrographic Conference

The American Association of Spectrographers is planning their 6th Annual Conference in Chicago, Illinois, May 6, 1955, on the subject "Industrial Applications of Spectroscopy." Contributed papers in the fields of Emission, X-Ray Fluorescence, or Absorption Spectroscopy as applied to industry are invited. Abstracts must be submitted by March 1, 1955. Please address all inquiries to: F. E. Stedman or E. E. Stilson, *Co-Chairmen*, Engineering Research Laboratory, Bendix Products Division, Bendix Aviation Corporation, 401 North Bendix Drive, South Bend 20, Indiana.

Shock Waves in Gases*

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This paper is in part a review but it also contains some original work. It deals with the manner in which shock waves are formed from finite compressions in gases and describes something of the structure of the shock front itself. The principal features of the behavior of shocks in reflection, refraction, and diffraction are discussed with particular attention given to anomalous observations and comparison with simple theories. Some results are given for shocks in real molecular gases showing relaxation effects. Quite a number of illustrations are included from the authors' own observations in the shock tube. The treatment is not exhaustive but covers many points likely to be of interest to teachers of physics.

INTRODUCTION

THE purpose of this paper will be to discuss some of the properties and effects of shock waves and their relation to sound waves. Everyone knows that the speed of sound in a gas is that speed at which small compressions are propagated by collisions between molecules. In fact, the speed of sound is approximately equal to the average molecular velocity. Shocks, on the other hand, are characterized by a large increase in pressure within a few mean free paths and advance at supersonic speeds. Explosions and lightning are two familiar sources of shocks. In both cases a region of high pressure is suddenly created from which a shock travels outward through the surrounding air. Since energy must be spread over an ever increasing surface as the shock expands its strength will diminish both from the expansion and from viscous dissipation. Eventually the shock will decay into a sound wave.

We shall first describe some of the properties of small amplitude sound waves. An extension of the theory then leads to the prediction that large amplitude waves tend to steepen on the front side and decline in back. When viscous effects are sufficiently small theory shows that the front of a continuous compression wave will eventually become infinitely steep. In the physical sense we say that a shock has formed. We shall then develop various relations pertaining to shocks and discuss their behavior in reflection, refraction, and diffraction. A device

called the shock tube for producing them in the laboratory under controlled conditions will be described. Finally, mechanisms for the eventual decay of shocks into sound waves are considered.

PROPAGATION OF SOUND WAVES

The speed of sound in a gas may be found by applying the principles of conservation of mass and momentum to the medium and assuming that the compressions produced by passage of the sound are isentropic.¹ A wave equation for the variation in pressure results,

$$a^2 \partial^2 p' / \partial x^2 = \partial^2 p' / \partial t^2. \quad (1)$$

Here p' is the change in pressure from ambient at a point x and time t and a is the velocity of sound given by

$$a^2 = dp/d\rho, \quad (2)$$

where it is understood that the derivative of pressure p with respect to density ρ is to be taken with the entropy constant. Many gases behave like an ideal gas for which

$$p = \rho RT, \quad (3)$$

R is the gas constant per unit mass, and T the temperature. If the specific heats C_p and C_v are constant and their ratio denoted by $\gamma = C_p/C_v$ the speed of sound may be expressed as

$$a^2 = \gamma RT. \quad (4)$$

This equation also gives the speed of a finite rarefaction wave advancing into a gas at rest,²

¹ See for instance W. F. Durand, *Aerodynamic Theory* (Verlag Julius Springer, Berlin, 1934), Vol. III, p. 210.

² I. I. Glass, *J. Aeronaut. Sci.* 19, 286 (1952).

* Part of the work described in this paper was supported by an U. S. Office of Naval Research contract.

TABLE I. Velocity of sound in ft/sec at 20°C for air, argon, carbon dioxide, and nitrous oxide obtained by four methods: (1) continuous sinusoidal waves, (2) extrapolation of weak shock velocities, (3) finite rarefactions, and (4) calculation from Eq. (4). In addition the values of γ at 20°C computed from the specific heats are included.

Gas	Air	Ar	CO ₂	N ₂ O
1. C. W.*	1126	1048	877	
2. Weak shock	1126		874	875
3. Rarefaction*	1126	1045	874	
4. $\sqrt{(\gamma RT)}$	1126	1046	877	872
5. γ	1.400	1.667	1.290	1.275

* Values for the velocity of continuous sound waves from the literature and the speed of a rarefaction wave are given in reference 2 together with probable errors. The data for weak shocks are original, and the estimated accuracy is ± 1 for air and ± 2 for the other gases.

predicts the asymptotic speed of a shock as the shock becomes vanishingly weak, and applies to gases with variable specific heat as long as Eq. (3) is an adequate equation of state. All of these methods have been used to determine the speed of sound in several different gases. It is interesting to compare the various experimental results with one another and with the theoretical value. Table I summarizes the data for four common gases, air, argon, carbon dioxide, and nitrous oxide. The agreement is quite gratifying in view of the variety of methods involved.

An appreciation of the extremely small amplitudes associated with the propagation of sound may be gained by noting that the pressure fluctuations in the loudest sound that is not actually painful is one thousandth of atmospheric pressure. The faintest sound the ear can detect is a variation of about 3×10^{-10} atmosphere pressure. Since the pressure amplitude of a spherically expanding wave drops off only inversely with distance, it is easy to see why sounds may be heard at great distances from their origin. Of course, accidental focusing by wind and temperature gradients may greatly increase this range.

In spite of the small amplitudes of audible sound waves there is a tendency for the crests to move slightly faster than the troughs since they are at a slightly higher temperature. Thus a sinusoidal wave will become distorted after traveling some distance. Viscosity acts to limit such distortion and a steady condition is reached in which the two effects just balance one another.

This problem has been studied in some detail.³

³ R. D. Fay, J. Acoust. Soc. Am. 3, 222 (1931).

When the simplifying assumption of infinitesimal amplitudes is dropped and the existence of viscous forces taken into account, a more complicated differential equation than the wave equation results whose solution may be expressed in terms of a Fourier series. The stable wave shape even in a one-dimensional problem is found to be a function of the amplitude. There is therefore no fixed wave shape but a continuously varying one leading finally to a sine wave when most of the energy has been dissipated. As an example the wave shape for a thousand-cycle plane wave with $p' = 10^{-5}$ atmosphere has been computed numerically⁴ and is shown in

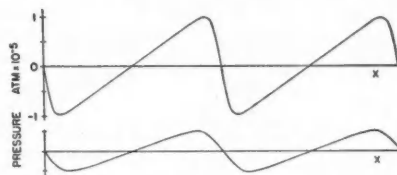


FIG. 1. Calculated wave shape for a thousand cycle note having an amplitude of 10^{-5} atmos and the same wave when the amplitude has decreased by half.

Fig. 1 along with the shape of the same wave when its amplitude has decreased by half.

FINITE COMPRESSION PULSES

A single compression wave of finite size may be generated in an imaginary experiment as follows: a tight-fitting piston in a long horizontal tube is accelerated to the right for a short time and then brought to rest again a distance L from its original position. A compression wave will advance through the air ahead of the piston and move every particle a distance L to the right. Initially the wave will have some shape as shown in Fig. 2a. Since each part of the wave advances with the local velocity of sound the same sort of steepening will occur with time as did in the large amplitude sound waves. For the moment let us neglect the influence of dissipation and follow the changes in wave shape arising from temperature effects alone.

Riemann⁴ studied this problem many years ago and found that the disturbance travels in such a way that any point B with pressure p and particle velocity v advances with the speed

⁴ B. Riemann, Göttingen Abhandlungen 8, 43 (1860).

$v+a$, where a is the local speed of sound. In terms of the speed of sound a_1 ahead of the compression B travels with speed $a_1 + (\gamma+1)v/2$.⁵ If one has the velocity profile at a given time $t=0$ (Fig. 2b), then the profile at a later time Δt may be found by advancing each point of the wave front a distance $[a_1 + (\gamma+1)v/2]\Delta t$. This construction has been carried out in Fig. 2c. It is evident that after a time $t=2/[(\gamma+1)(dv/dx)_{t=0}]$ a vertical tangent will be reached as in Fig. 2d.

An experimental verification of the foregoing theory has been obtained in the shock tube with the aid of an interferometer. Three pictures in Fig. 3 show how the density in a compression wave varies with time. Suffice it to say for the moment that the vertical displacement of a given fringe is directly proportional to the density. We conclude that an infinite slope in the theoretical solution corresponds to a shock in

shock. Then the equations are:

conservation of mass

$$\rho_1 v_1 = \rho_2 v_2; \quad (5)$$

momentum

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2; \quad (6)$$

energy

$$h_1 + v_1^2/2 = h_2 + v_2^2/2. \quad (7)$$

In general the enthalpy is a fairly complicated function of temperature so that these three equations and the equation of state are difficult to solve. For the special case of an ideal gas with constant specific heat, however, $h = c_p T = \gamma p / \rho(\gamma-1)$. If the velocities are eliminated an equation between the pressure and density ratios across the shock is obtained,

$$\frac{\rho_2}{\rho_1} = \left[1 + \frac{\gamma+1}{\gamma-1} \frac{p_2}{p_1} \right] / \left[\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1} \right]. \quad (8)$$

This relation was found independently by Rankine and Hugoniot in the last century⁶ and is named after them. It is applicable to monatomic gases over a wide range of temperatures and works well for air up to about 500°K. Above this range appreciable vibrational energy is present so c_p is no longer constant.

The symmetry of Eq. (8) suggests that rarefaction shocks might also exist in nature. These may be shown to lead to a decrease in entropy,

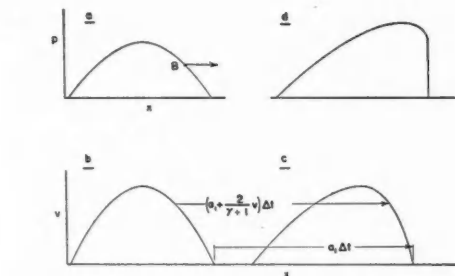


FIG. 2. Construction showing how a compression wave steepens into a shock according to Riemann's theory. When a vertical tangent is reached we say that a shock is formed.

real life. In the next section we shall investigate the properties and structure of shocks by means of the conservation equations.

SHOCK WAVES

The conservation equations for a fluid enable us to draw many conclusions about the behavior of shocks without knowing anything about their structure. For an observer riding on a shock in one-dimensional steady flow let us use the symbols p_1 , v_1 , ρ_1 , and h_1 to give the values of pressure, velocity, density, and enthalpy at a point far enough ahead of the shock for conditions to be uniform. Similarly the subscript 2 will denote equilibrium conditions behind the

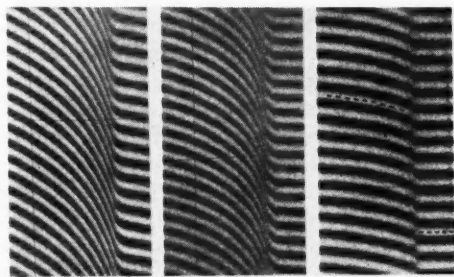


FIG. 3. Fringe pattern obtained with an interferometer of three stages in the formation of a shock from a continuous compression. The wave is traveling toward the right. The vertical position of a given fringe is proportional to the density at that point. In the last picture a jump of seven fringes occurs.

⁵ W. J. M. Rankine, Phil. Trans. Roy. Soc. 160, 277 (1870). H. Hugoniot, J. école polytech (Paris) Nos. 57-59 (1887-89).

⁶ Reference 1, Vol. 3, p. 216.

however, and must be ruled out by the second law of thermodynamics.⁷

If velocities are measured in units of the local speed of sound several useful relations assume a specially simple form. With the same assumptions as before and the Mach number M defined as v/a , the shock density and pressure ratios are

$$\rho_2/\rho_1 = 1 / \left[\frac{2}{(\gamma+1)M_1^2} + \frac{\gamma-1}{\gamma+1} \right], \quad (9)$$

$$p_2/p_1 = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1). \quad (10)$$

Both are seen to be monotonically increasing functions of the shock speed. The speed becomes infinitely great for an infinite pressure ratio but the density ratio approaches a finite value of $(\gamma+1)/(\gamma-1)$, which is 4 for the monatomic gases.

The Mach number of the flow leaving the shock can be derived from the preceding relations and is given by

$$M_2^2 = \frac{v_2^2}{a_2^2} = \frac{1 + \frac{\gamma-1}{\gamma+1}(M_1^2 - 1)}{1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)}. \quad (11)$$

Since M_1 is always greater than one for a shock to exist the numerator is always less than the denominator and we may conclude that v_2 is

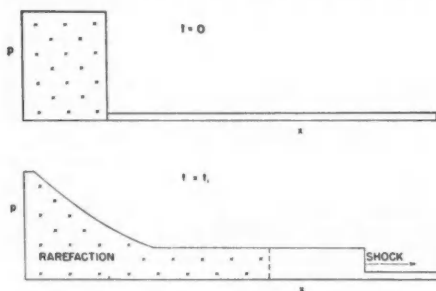


FIG. 4. Schematic drawing of the pressure distribution in a shock tube before and just after the diaphragm is burst. As an aid in visualizing the flow produced, the volume occupied by particles originally in the high pressure end is cross-hatched.

⁷ H. Liepmann and A. Puckett, *Aerodynamics of a Compressible Fluid* (John Wiley and Sons, Inc., New York, 1947), p. 40.

always less than a_2 . This property is essential to the stability of a shock with respect to slight disturbances for it will be able to catch any signal that may run ahead and will also be caught by any that get behind.

THE SHOCK TUBE

To investigate the properties of these shocks it is desirable to be able to produce them at will in the laboratory. A device for doing this is called the shock tube. It consists of a long pipe divided into two sections, one of which at the time of initiation is at high pressure and so propagates a compression into the other which rapidly becomes a shock. The high-pressure region may be created in several ways. It may be built up mechanically behind a diaphragm which is broken at the moment of initiation. Or a large change in pressure may be induced by such means as chemical explosion, electric discharge or exploding wires. The shock strength is controlled by the starting pressure and temperature ratios.

Once the shock is formed it moves with constant velocity through the medium ahead and is followed by a column of gas of uniform state traveling with velocity $v_1 - v_2$ relative to the laboratory. The pressure distribution before and shortly after a cellophane diaphragm is punctured is shown in Fig. 4. The flat-topped shock will persist until it reaches the end of the tube or is caught by the rarefaction which sweeps into the high pressure gas and is reflected from the back end of the tube. Rather complete details on the theory of the shock tube may be found in the literature.⁸⁻¹³

The Princeton shock tube is 4 in. by 18 in. in cross section and 38 ft long. Experience has

⁸ Bleakney, Weimer, and Fletcher, *Rev. Sci. Instr.* **20**, 807 (1949).

⁹ J. Lukasiewicz, "Shock tube theory and applications," National Aeronautical Establishment of Canada, Report 15, 1952.

¹⁰ P. W. Geiger and C. W. Mautz, "The shock tube as an instrument for the investigation of transonic and supersonic flow patterns," Engineering Research Institute, University of Michigan, 1949.

¹¹ Glass, Martin, and Patterson, "A theoretical and experimental study of the shock tube," University of Toronto Institute, Aerophysics Report 2, 1953.

¹² R. K. Lobb, "On the length of a shock tube," University of Toronto Institute of Aerophysics Report 4, 1950.

¹³ Resler, Lin, and Kantrowitz, *J. Appl. Phys.* **22**, 878 (1951).

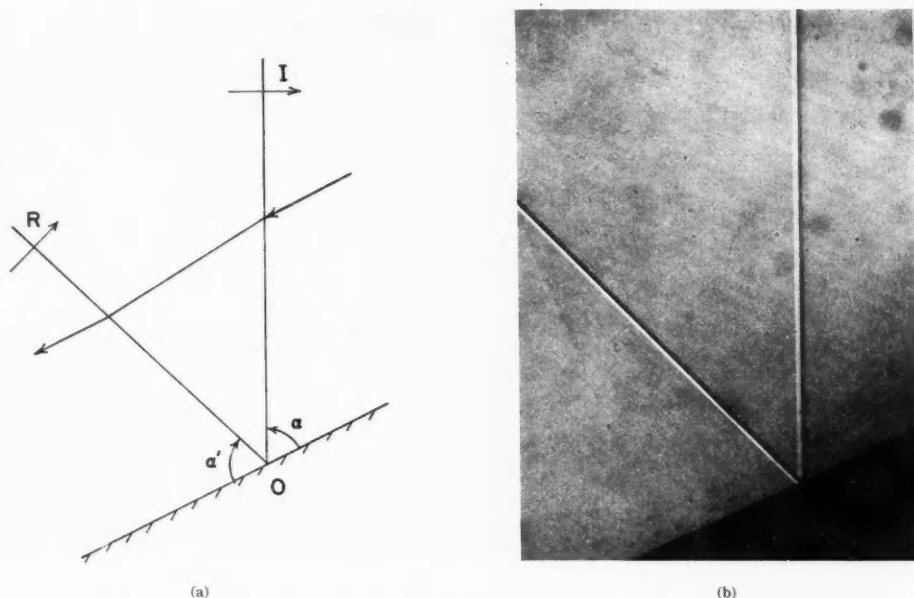


FIG. 5. Shadow picture showing reflection of a shock of Mach number 1.034 from a solid wall. The incident shock sets the air in motion. Relative to the point O a given particle follows the path shown in the construction on the left.

shown that the shock has sufficient time to become fully formed after traveling about ten tube diameters. Shock velocities are measured by noting the time of passage between two schlieren light screens a known distance apart. As the shock passes each screen the beam is momentarily deflected into a photocell. The pulses from the photocells are fed into an oscilloscope and recorded photographically on a drum camera. The pulse from the second light screen also actuates an adjustable electronic delay circuit which triggers a one microsecond spark. Since the shock strength is very accurately repeatable from one shot to the next a series of pictures may be obtained showing the development of any event.

The flow patterns may be observed by interferometer, schlieren, or shadow photography. Shadow pictures are the easiest to get and give the clearest information on the position of shocks. For quantitative information an interferometer is superior since the shift of fringe position is directly proportional to the density change in two-dimensional flow. By geometrical optics it can be shown that schlieren gives a

measure of density gradient while shadow is sensitive to the second derivative of density. As an example the shadow picture in Fig. 5 and interferograms in Fig. 6 taken with white and monochromatic light show a shock of Mach number 1.034 being reflected from an incline. Careful measurements reveal that the apparent shock thickness in Fig. 5 is a result of the 1- μ sec spark duration measured with a rotating mirror. In a later section we shall see that the actual thickness is $\sim 10^{-5}$ inch. The small number of fringes visible in the white light picture enable us to identify fringes on either side of the shock. If the shift in fringe position δ is measured in units of the original fringe spacing, the density change is given by Eq. (12) where λ is the wavelength of light, n_s is the index of refraction and ρ_s the density of the gas at STP, and l is the width of the test section,

$$\rho_2 - \rho_1 = [\lambda \rho_s / l (n_s - 1)] \delta. \quad (12)$$

Since the interferometer and the light screens provide two independent methods of measuring shock strengths we may make an experimental check of the Rankine-Hugoniot relation [Eq.

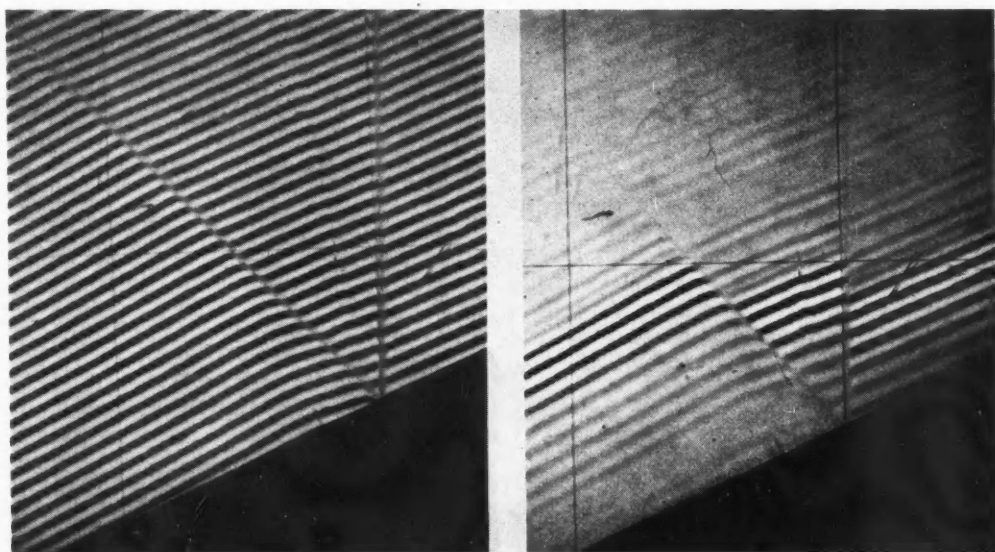


FIG. 6. Interferograms taken with monochromatic and white light of the same shock shown in Fig. 5. The small number of fringes visible in the white light picture makes it possible to find the density change across a shock.

(8)], or, as some prefer to look at it, a check on the accuracy of our measurements. Shock pressure ratios for air have been computed from interferometer data using Eq. (8) and from velocity measurements using Eq. (10) for a large number of experiments. In each of nine arbitrary intervals between $p_1/p_2 = 0.95$ and 0.15 the average values agree to one part in a thousand, corresponding to relative accuracies of 0.1 percent for weak shocks and 0.7 percent for the stronger ones. Above this range of shock strengths, $M > 2.4$, the temperature becomes high enough to excite appreciable molecular vibrational motion in O_2 and N_2 so that γ is no longer constant. Solution of the flow equations for such cases will be discussed later.

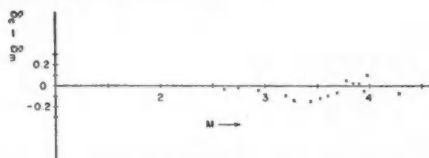


FIG. 7. Direct experimental test of the Rankine-Hugoniot relations for argon. The calculated fringe shift δ_c is obtained by these relations from the shock velocity. The difference between the resulting value and that actually measured with the interferometer, δ_m is plotted as a function of shock Mach number.

Argon, on the other hand, has such a large electronic excitation energy that its behavior should agree with prediction of the present theory up to very high temperatures. Accordingly, a series of pictures have been taken in argon with shock Mach numbers between 2.6 and 4.3. The large pressure ratio required to generate the strongest shocks limits the pressure in the channel to about 10 mm-Hg. The corresponding fringe shift is slightly less than two so that measurements with the interferometer become decreasingly sensitive. A meaningful comparison of speed and fringe measurements may be made by computing the fringe shift we would expect from the shock velocity. This has been done for fifteen experiments and the results are plotted in Fig. 7. Here the fringe shift computed from the velocity minus that actually measured is shown as a function of Mach number. The average of all values is $-1/20$ th fringe and it may be seen that no point differs from this average by more than $1/15$ th of a fringe. This is well within our estimated error from all causes which we have previously taken as $1/10$ th fringe. So far as we know these results in air and argon are the best direct experimental verification of the validity

of the assumptions made in deriving the Rankine-Hugoniot relation. It is, of course, just the result we expected to get.

SHOCK INTERACTIONS

Probably the simplest problem of shock interaction that one can formulate is that analogous to Snell's law for reflection and refraction. For shocks the angle of incidence and reflection will in general not be equal because the incident shock produces an increase of entropy of the gas so that the reflected shock must travel through a medium having a different thermodynamic state. The problem of reflection on a wall may be formulated as follows: For a given shock strength and angle of incidence what reflected waves will leave the gas flowing parallel to the wall? In Fig. 5 the incident shock I strikes the wall at an angle α to produce a reflected shock R at the angle α' . Consider a coordinate system in which the point of intersection O is at rest. The change in components of velocity normal to the shocks can be found from Eqs. (9) and (11), while the tangential components of velocity are unaffected. Polachek and Seeger¹⁴ solved the

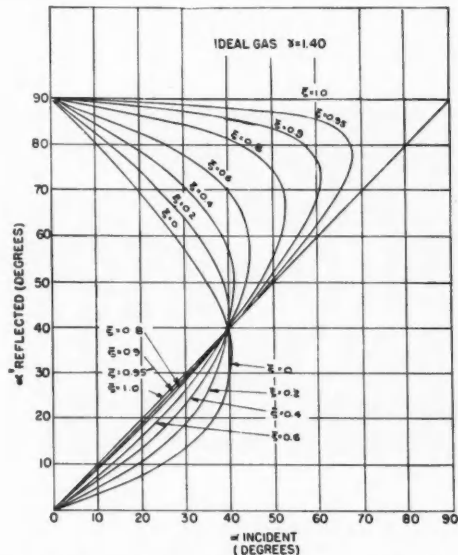


FIG. 8. Theoretical solution for the reflected shock angle α' as a function of the angle of incidence α and the shock strength $\xi = p_1/p_2$.

¹⁴ H. Polachek and R. Seeger, "Regular reflection of shocks in ideal gases," Explosive Research Report No. 13, Bureau of Ordnance, U. S. Navy Dept., 1944.

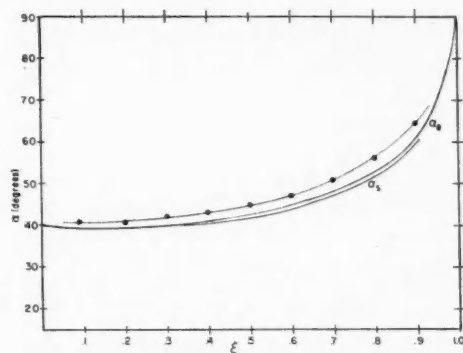


FIG. 9. Comparison of theory and experiment for the transition between two-shock (regular) reflection and three-shock (Mach) reflection. α_e is the angle above which no two-shock solution exists in Fig. 8, α_s is the angle of incidence for which the outflow from the reflected shock becomes sonic, and α_e is the experimentally observed transition angle (upper curve).

resulting set of equations for α' subject to the condition that the two shocks deflect the flow equal amounts but in opposite directions. Their results are shown in Fig. 8 for eight shock strengths. The parameter ξ is defined as the ratio of pressure ahead of a shock to that behind, $\xi = p_1/p_2$, so that $\xi = 1$ gives the sonic case and $\xi = 0$ corresponds to an infinitely strong shock. Two features of these curves are of special note: only for sufficiently small values of α , $\alpha < \alpha_e$, do any solutions exist and below these extreme values two possible reflected waves can occur. Of the two, that for smaller α' gives the weaker reflected shock.

One further limitation is placed on the region of two-shock or regular reflection by the relations between the curvatures of shocks and streamlines where they cross.¹⁵ Above a limiting angle of incidence the reflected wave is curved throughout its length and difficulties are encountered in meeting the boundary conditions at the wall. As a consequence the flow behind the intersection must be at least sonic for the simple two-shock idea to apply. The angle for which the outflowing air becomes sonic with respect to O is designated by α_s in Fig. 9 where it is plotted as a function of ξ together with the extreme angle α_e .

Extensive experiments have been carried out

¹⁵ A. H. Taub, Ann. Math. 58, 501 (1953).

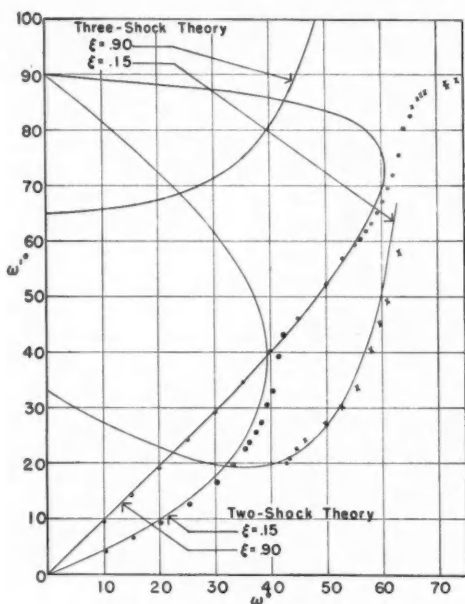


FIG. 10. Experimental and theoretical values for the reflected shock angle α' , or for Mach reflection ω' , as a function of the angle of incidence for one strong shock strength, $\xi=0.15$, and one weak shock, $\xi=0.9$. The circles indicate points where regular reflection is observed and the x's are for Mach reflection. Here, as is usual practice, the coordinates are $\omega=\alpha-\chi$ and $\omega'=\alpha'+\chi$ (see Fig. 12), where χ is zero for regular reflection. The angle between the incident shock and the incoming flow relative to the intersection is therefore ω .

by Smith,¹⁶ and his measurements show good agreement between experiment and the solution leading to the weaker reflected shock. Results for $\xi=0.15$ and 0.9 are shown in Fig. 10. It might be mentioned that in steady supersonic flow past a wedge the weaker shock is also the one commonly observed. No proof of greater stability for the weaker wave has been found as yet.

When the angle of incidence exceeds the value for which any regular reflection exists a new and quite different pattern is observed experimentally as shown in Fig. 11.¹⁷ Mach reflection,

¹⁶ L. G. Smith, "Photographic investigations of the reflection of plane shocks in air," OSRD Report No. 6271, 1945.

¹⁷ The interferogram shown in Fig. 11 was obtained using a different adjustment of the interferometer from that for Fig. 6. In this case the mirrors are set exactly parallel to give uniform interference over the entire field when no disturbances are present. When the shock arrives regions of equal density will change the optical path length equally and appear as light and dark fringes. Thus the density field is presented directly. This method has

named for Ernst Mach who first observed the effect,¹⁸ shows the incident and reflected shock meeting some distance from the wall with a third shock extending from the point of intersection to the wall. This third shock will be called the Mach stem and the point of intersection the triple point.

An additional discontinuity extends back from the triple point. This is a slipstream separating the gas which has passed through the incident and reflected shocks from that which has gone through the Mach stem. The entropy changes are slightly different so that the requirement of equal pressures if the streamlines are to be parallel gives slightly different particle velocities. The magnitudes are such that the gas above flows away from the triple point faster than that below.

The use of a symmetrical wedge rather than an incline on a plane reduces the disturbing effect of the viscous boundary layer at the solid surface. Since there is no dimension inherent in the configuration it seems reasonable to suppose, and experiment bears this out, that the pattern grows similar to itself in time. That is, the flow is pseudostationary and the three independent coordinates x, y, t reduce to two:

$t \rightarrow$

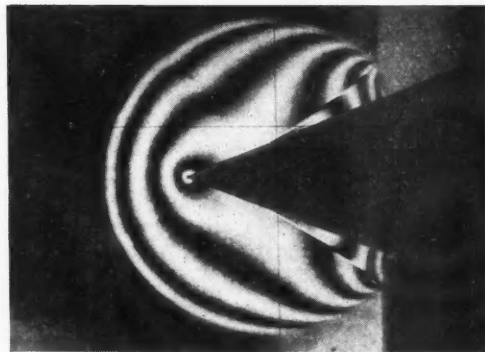


FIG. 11. Mach reflection of a shock from a symmetrical wedge. $\xi=0.42$. For this picture the interferometer is adjusted so that a uniform field exists when no disturbances are present. Contours of equal brightness then become lines of constant density.

the disadvantage that the contours cannot be labeled without doing additional experiments but will be used here frequently because it illustrates features of the flow so vividly.

¹⁸ E. Mach, Akad. Wiss. Wien 77, 819 (1878).

x/t , y/t . This was first suggested by von Neumann.¹⁹ Even with this reduction in the number of variables the mathematical problem is so formidable that no general solution for the flow field in Mach reflection has been discovered. Several proposals have been put forth to treat special cases and a representative example will be discussed in the following paragraphs.

If the flow is pseudostationary the triple point T travels along a straight line through the corner C making some angle χ with the surface as in Fig. 12. By letting the flow proceed long enough any degree of magnification desired of the region near the triple point can be obtained. No singularities in shock curvature have been observed in the pictures. This suggests that conditions are continuous in all of the angular domains around T bounded by the four discontinuities, and it seems plausible to assume that the shocks can be treated as straight as one approaches arbitrarily close to T . To an observer riding with the triple point the three-shock situation is thus similar to the two-shock problem investigated before. Here the condition of flow parallel to the wall is replaced by the requirement that the velocities behind the triple point be parallel and the pressures equal. von Neumann has solved the resulting equations in terms of the shock angles $\omega = \alpha - \chi$ and $\omega' = \alpha' + \chi$ relative to the line at angle χ . The resulting values of the three-shock solution are plotted in Fig. 10 (where for regular reflection $\chi = 0$ and $\alpha = \omega$) for $\xi = 0.15$ and 0.9. Agreement with experiment is fair for the strong shocks ($\xi = 0.15$) but is poor for the weak case. This result is surprising in view of the success of the two-shock theory. An additional problem is that of understanding the persistence of two-shock reflection beyond the theoretical limit as illustrated in Fig. 9. The curve α_0 shows the experimental points where χ becomes zero, representing the transition between Mach and regular reflection. In summary, we may say that we have a local theory for the angles at the intersection which work for regular reflection below α_0 and for Mach reflection above α_0 for strong shocks. Outside this range clearcut discrepancies exist which still need to be explained.

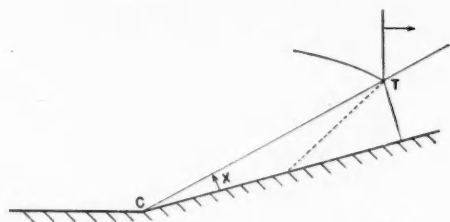


FIG. 12. Construction showing how the angle χ is defined. When the flow is pseudostationary the triple point T moves in such a way that χ remains constant.

The only complete solutions of the reflection problem discovered so far are restricted either to nearly glancing incidence ($\alpha = 90^\circ$) or to nearly head on reflection ($\alpha = 0^\circ$). Since experiments must be made at an angle at least 5° different from these limits in order to detect anything a problem of interpretation arises which can be resolved either by extrapolating experimental data to zero angle or by making a second-order correction to the theory. Four papers²⁰⁻²³ dealing with solutions of this type agree quite well with experiment.^{24,25} One such case will be reviewed here.

Ting and Ludloff²³ have reported a method which they used to find the flow field when an incline is struck by a shock strong enough to produce supersonic flow relative to the corner. (For air this requires that p_2/p_1 be greater than 5.) The mathematical approach used has been explained more fully elsewhere.²⁴ White²⁵ has compared their results for one shock strength with his measurements. Figure 13 shows the theoretical and experimental results for a shock of strength $\xi = 0.137$. Evidently there is good agreement in both the shape of the pattern and the quantitative values of the density contours. Such discrepancies as exist can be ascribed to the finite angle used in the experiment.

Knowledge about shock refraction is in a very

²⁰ V. Bargmann, "On nearly glancing reflection of shocks," OSRD, No. 5171, 1945. See also reference 24.

²¹ M. J. Lighthill, Proc. Roy. Soc. (London) **A198**, 454 (1949).

²² M. J. Lighthill, Proc. Roy. Soc. (London) **A200**, 554 (1950).

²³ L. Ting and H. F. Ludloff, J. Aeronaut. Sci. **18**, 143 (1951).

²⁴ Fletcher, Taub, and Bleakney, Revs. Modern Phys. **23**, 271 (1951).

²⁵ D. R. White, "An experimental survey of the Mach reflection of shock waves," Proc. Second Midwestern Conference on Fluid Mechanics, Ohio State University, 1952.

¹⁹ J. von Neumann "Oblique reflection of shocks," Exptl. Res. Rep. No. 12, Bureau of Ordnance U. S. Navy Department, 1943.

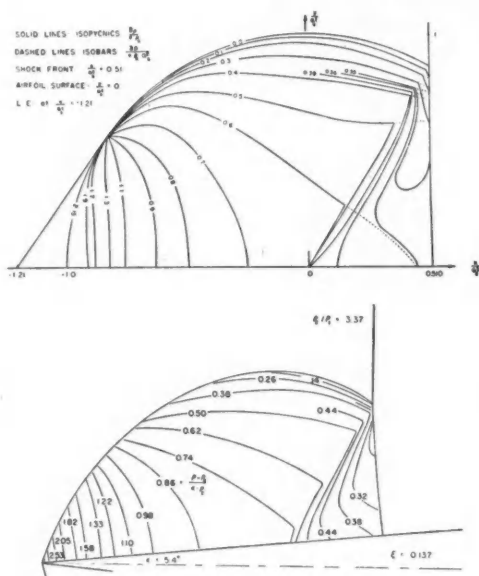


FIG. 13. Comparison of Ting and Ludloff's solution (above) with White's experimental result (below) for a shock of Mach number 2.53 striking a small incline of 5.4° . The solid lines are contours of constant density.

much more primitive state than for reflection because of the multiplicity of solutions obtained theoretically and experimental difficulties in forming a suitable interface between two gases. Again let us study "regular" refraction first where only the possibility of straight shocks is considered. In Fig. 14 the line OD represents the boundary between the two gases having specific heat ratios γ_1, γ_5 , and velocities of sound a_1, a_5 . The incident, reflected, and transmitted waves are represented by I, T , and R , respectively. Five quantities are necessary to specify a problem completely; the incident shock strength ξ , its angle of incidence α , the ratio of sound speeds a_1/a_5 , and γ_1 and γ_5 . Relative to the point of intersection O the boundary conditions are that the flow direction and pressure behind R be the same as behind T . The boundary may be deflected to some new direction OB . Solutions of the resulting equations for several combinations of gases have been obtained by Taub²⁶ and by Polachek and Seeger²⁷ with the aid of electronic computers. Polachek and Seeger also

considered the possibility that the reflection R be an expansion fan centered at O rather than a shock.

Several physical limitations must be placed on the mathematical solutions in order that the results be meaningful. Even when this is done, however, more than one choice often remains and one must apparently rely on experiment to pick the correct branch. A special argument may be made for those branches of the solution which approach the well verified acoustical version of Snell's law, $\sin \alpha / \sin \alpha'' = a_1 / a_5$, for vanishing shock strength and the head-on refraction of finite shocks where one-dimensional solutions may be found easily. Until experiments are carried out, however, such discussions must be of a purely speculative nature.

In addition to the existence of an extreme angle α_c beyond which no solutions of regular refraction exist at all, the following possibilities must be considered as α varies; (a) the flow velocity normal to the reflected wave must be at least sonic, i.e., α is limited by some value which we shall denote as α_s ; (b) a reflected shock may vanish and become a reflected rarefaction instead, the transition occurring at $\alpha = \alpha_t$; (c) for the transmitted shock to exist the velocity of the point of intersection must be supersonic with respect to the lower medium (gas 5), limiting α to values less than α_t . Extra complications will arise if the outflow is not supersonic in both media.

A single example will be included here; that of a shock striking an air-methane interface. For

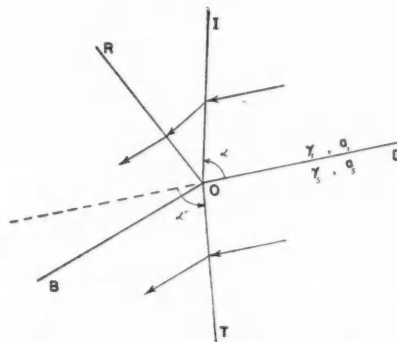


FIG. 14. Pattern of "regular" refraction at the interface OD between two gases. Flow directions relative to point O are shown.

²⁶ A. H. Taub, Phys. Rev. **72**, 51 (1947).

²⁷ H. Polachek and R. Seeger, Phys. Rev. **84**, 922 (1951).

other cases the reader is referred to the papers already cited. For normal incidence ($\alpha=0$) a shock is transmitted through the methane and a rarefaction reflected into the air. This occurs because the particle velocity behind the shock in methane is greater than that of the air passed over by the incident shock so the air must accelerate to the right in order that no vacuum forms at the interface. A rarefaction traveling to the left through the air accomplishes just this.

Theoretical investigation of the limiting angles previously enumerated give the data presented in Fig. 15 for α_s , α_t , and α_i as a function of ξ . For a given ξ a reflected rarefaction is predicted for all values of α from 0 to α_t . At $\alpha=\alpha_t$ no reflected wave occurs at all. Between α_t and α_i the reflected wave is a shock and regular three-shock

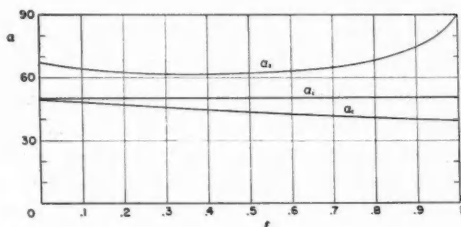


FIG. 15. Limiting angles on the regular refraction of shocks at an air-methane interface. For angles of incidence less than α_t theory predicts that the reflected wave is a rarefaction. Between α_t and α_i a reflected shock should occur, and above α_i no solution for regular refraction exists. In this case the requirement that the flow normal to the reflected wave R be supersonic is less severe than that given by α_i .

refraction should be observed. The interferogram shown in Fig. 16 verifies this prediction. When α_t is reached the outflowing methane becomes subsonic and the regular refraction pattern can no longer exist. Slightly beyond α_t the transmitted wave would run ahead and form some other pattern.

No theoretical approach has been made to such a situation so far but as may be seen from Fig. 17 the flow is quite complicated. At $\alpha=76^\circ$ and $\xi=0.86$ the transmitted wave is considerably ahead of the incident wave and a signal has been retransmitted from the methane to the air. The incident and reflected waves have the appearance of Mach reflection except that the Mach stem is moving into a nonuniform region. In this case of an air-methane interface, the

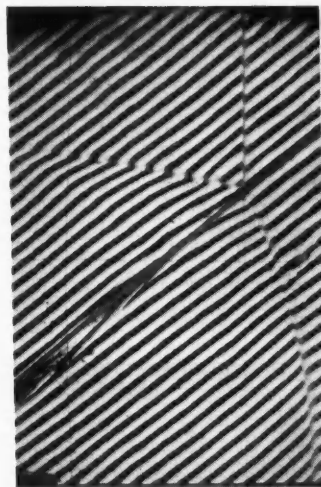


FIG. 16. Regular refraction of a shock at an air-methane boundary. $\xi=0.86$, $\alpha=51^\circ$. Incident, reflected, and refracted shocks are visible. In this picture a thin film separates the two gases. The film is deflected downward by the shocks and a small gap may be seen between the film and holder behind the point of intersection. The ground plane is visible at the bottom of the picture.

limitation imposed by α_s and α_e are not reached. For other combinations of gases quite different patterns may be observed.

Diffraction, the third kind of shock interaction we shall discuss, bears even less similarity to its optical counterpart than do reflection and refraction because shocks have no wave nature (in spite of the common usage in calling them

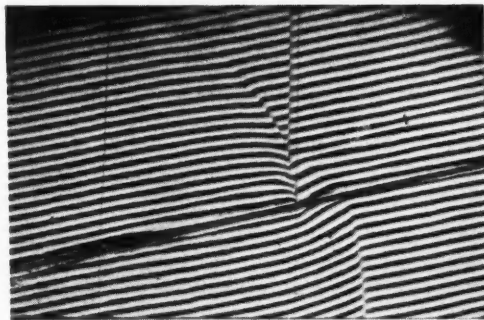


FIG. 17. Refraction at an air-methane interface with $\xi=0.86$, $\alpha=76^\circ$. Here α is greater than the limiting angle α_t so that the transmitted shock has run ahead of the incident shock and retransmitted a compression wave into the air. Except for this disturbance the incident and reflected shocks are very like a Mach reflection in appearance.

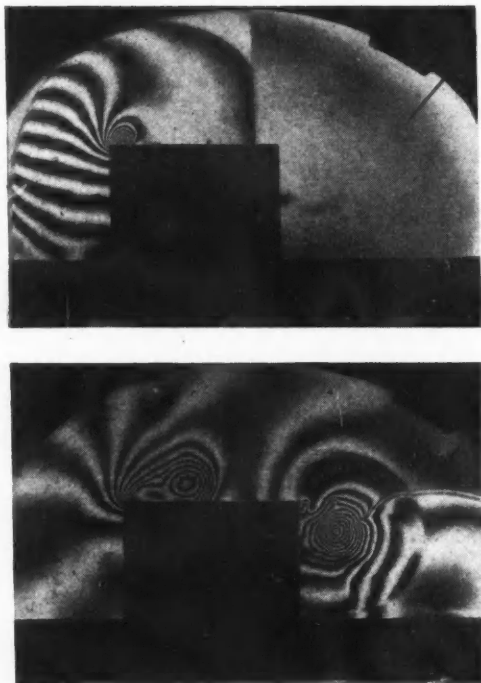


FIG. 18. Two stages in the diffraction of a shock over a rectangular block. The eddies formed behind each corner are regions of very low pressure. In the second picture the incident shock has reflected from the ground behind the block, entered the second eddy, been spun around clockwise, and come out as a nearly cylindrical shock.

shock waves) and therefore exhibit little of the phenomena of wave interference. Shock diffraction is important from the practical viewpoint these days for the engineer faced with the problem of the design of blast resistant structures. Not only is a structure suddenly immersed in an atmosphere of increased pressure when struck by a large blast wave but the shock front at the leading edge of the oncoming disturbance is diffracted about the object in a complicated manner setting up cross currents and eddies which profoundly affect the load distribution on the structure. Figure 18 shows two stages in the diffraction of a shock over a two-dimensional block in the shock tube. In the first picture the incident shock may be seen crossing the top of the block. A reflected shock travels to the left to inform the oncoming stream of the object's presence. An eddy forms behind the corner and

is gradually swept downstream as it increases in size. The second shot shows how the incident wave reflects from the plane and is caught up in the eddy behind the block. Since a large clockwise circulation exists there the shock is spun around and emerges as a cylindrical shock. The interferogram may be evaluated in terms of the density and from these data the pressure can be calculated everywhere with considerable accuracy.^{28,29} The pressures at the surface of the block give, of course, the force distribution (neglecting viscous effects) over the entire obstacle at this particular time. A series of such pictures allows the loads to be determined as a function of time. As the transient effects disappear one would expect the pressure loading on the block to approach that observed in wind tunnels. This is indeed the case.

Several other examples of diffraction are shown in Figs. 19, 20, and 21. There is no adequate theory for such phenomena. A considerable amount of experimental data has therefore been collected giving information about the pressure loading on a wide variety of objects.^{28,29}

Space does not permit a discussion of all shock-wave interaction phenomena. For instance a considerable amount of work has been done on one-

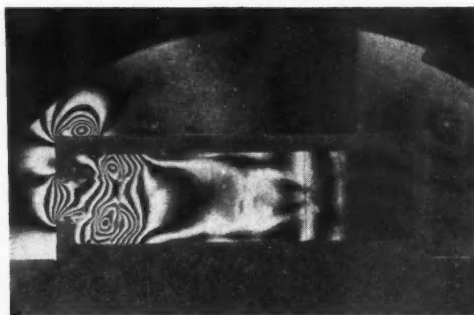


FIG. 19. Early stages in the filling of a hollow block after it has been struck by a shock. The net force on the top of the block is downward; later an upward force is observed when the shock inside reflects from the back wall.

²⁸ Bleakney, White, and Griffith, *J. Appl. Mech.* 17, 439 (1950).

²⁹ W. Bleakney, "A shock tube investigation of the blast loading of structures," *Proceedings of the Symposium on Earthquake and Blast Effects on Structures*, University of California, Los Angeles, p. 46, 1952.

dimensional interactions.¹¹ Here theory and experiment are in good accord.

SHOCK STRUCTURE

A further goal in fluid dynamics is to understand the mechanisms that determine the thickness and structure of a shock. One might ask, for instance, whether the equations of a continuous fluid apply or not. If they do then viscosity and heat conduction are certainly important in the theory of shock structure because velocity and temperature gradients must be very high within the shock. The conservation equations for one-dimensional steady flow assume the form

$$(d/dx)\rho v = 0, \quad (13)$$

$$\frac{d}{dx}\rho v^2 + \frac{dp}{dx} - \frac{4}{3}\mu \frac{d^2v}{dx^2} = 0, \quad (14)$$

$$\rho v \frac{d}{dx} \left(h + \frac{v^2}{2} \right) = \frac{d}{dx} \kappa \frac{dT}{dx} + \frac{2\mu}{3} \left[2 \left(\frac{dv}{dx} \right)^2 - v \frac{d^2v}{dx^2} \right], \quad (15)$$

where μ and κ are the coefficients of viscosity and thermal conductivity, respectively. Becker³⁰ first succeeded in solving the equations but made the assumption that these two coefficients are constant. Thomas³¹ pointed out that Becker's result must be incorrect for strong shocks where the dependence of μ and κ on temperature is im-

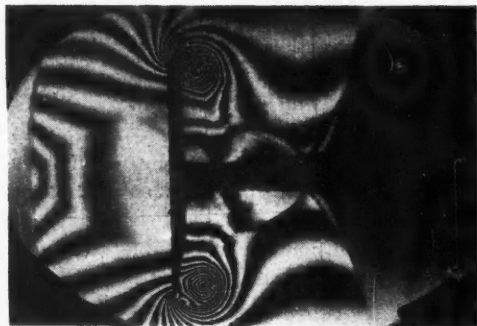


FIG. 20. Diffraction past a thin plate mounted perpendicular to the direction of shock travel. In this experiment one end of the plate was ground to a sharp edge to see what change this would make in the pattern. Very little effect can be seen.

³⁰ R. Becker, Z. Physik 8, 321 (1923).

³¹ L. H. Thomas, J. Chem. Phys. 12, 449 (1944). See also A. E. Puckett and H. J. Stewart, Quart. Appl. Math. 7, 457 (1950).

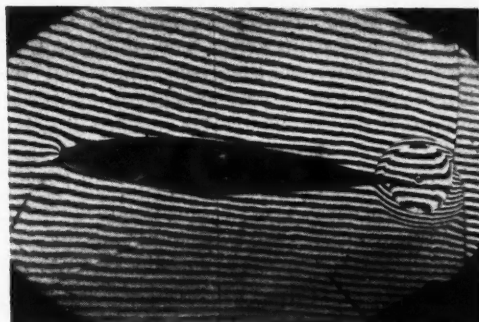


FIG. 21. A weak shock passing over an airfoil at a small angle of attack. The shock below the wing arrived at the rear first. Thus the diffracted part above the tail and the rarefaction which started downward simultaneously are a bit ahead of the rarefaction and shock, respectively, formed when the shock above the wing arrived at the rear. A small vortex may be seen a short distance behind the trailing edge.

portant. Using the kinetic theory result that both μ and κ vary as \sqrt{T} , Thomas obtained shock thicknesses considerably greater than those from the Becker theory. Nevertheless, since the values found are only a few mean free paths for shocks faster than $M \sim 1.2$, continuous fluid theory may not give as satisfactory an interpretation as the kinetic theory of gases.

Two rather distinct lines of approach have been taken in applying the kinetic theory to shocks. In one, successive approximations to a Maxwell-Boltzmann distribution

$$f_0(v_x, v_y, v_z) = (m/2\pi kT)^{3/2} \times \exp[-m(v_x^2 + v_y^2 + v_z^2)/2kT] \quad (16)$$

of molecular velocities are made by taking into account the effect of nonuniform conditions along the x direction.³²

$$f = f_0 + f_1 + f_2 + \dots \quad (17)$$

Each term is assumed to be a function only of those preceding it.

$$f_1 = f_1(f_0), \quad f_2 = f_2(f_0, f_1), \\ f_3 = f_3(f_0, f_1, f_2), \dots$$

Equations (13)–(15) may be derived by retaining only $f_0 + f_1$. Thus a calculation using only the

³² For a discussion of this method see S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, Cambridge, 1939), Chap. 7.

first-order terms would give just the Thomas theory again. Further terms from the expansion presumably improve the theory provided the solution converges rapidly enough to justify the assumption that the molecular velocity distribution may be obtained from Eq. (17) at all. Zoller³³ has carried out calculations to the third order on the variation of properties through shocks of pressure ratio $p_2/p_1 = 1.5, 4, \text{ and } 6.5$. For the strongest shock $p_2/p_1 = 6.5$, the third-order terms are comparable in size to the first and second so that the accuracy of this solution is still in question. For $p_2/p_1 \leq 4$ Zoller's results appear to be satisfactory.

Since shocks appear to get thinner with increasing Mach number Mott-Smith³⁴ has made the interesting suggestion that the velocity distribution may be composed of a mixture of two Maxwellian distributions corresponding to uniform conditions on the high- and low-temperature sides of the shock. Appreciable numbers of molecules from these two populations penetrate to the center of the shock. The structure itself is found by solving Boltzmann's equation³² for the transport of kinetic energy across the shock.

A comparison of the various theories may be made by giving the shock thickness L as a function of Mach number. The quantity L is defined by the construction in the insert in Fig. 22 where it is seen to be the distance between intercepts of a tangent drawn at the steepest point of the shock profile. Very little difference exists in the

shape of this profile in the various theories so no attempt is made to distinguish among them in comparing thicknesses. The parameter chosen as ordinate in Fig. 22 is Maxwell's mean free path λ in the gas ahead of the shock divided by shock thickness L .

Owing to the extreme thinness of shocks only a little experimental data can be reported so far. Cowan and Hornig³⁵ have devised a method of measurement based on the reflectivity of a shock front. The amount of light reflected depends on the wavelength of the light, its angle of incidence, the index of refraction of the gas, the shock thickness, and its density profile. Measurements have been made in argon and nitrogen up to $M = 2$.³⁶ The experimental points for argon are plotted on Fig. 22. Apparently the Mott-Smith and Zoller theories come closest to these experiments.

When molecules have rotational and vibrational energy as well, the picture becomes more complicated for, as has been known for a long time, exchange of kinetic energy with internal degrees of freedom takes place rather slowly. In nitrogen, for example, the average number of collisions to attain rotational equilibrium is slightly larger than the number in a weak shock front so that the foregoing theories are not directly applicable. In carbon dioxide at STP about twenty collisions are required on the average to exchange rotational quanta while 80 000 are needed for vibration. One might assume in this case that the shock consists of a narrow zone in which translation and rotation reach equilibrium followed by a region where vibrational energy comes slowly into adjustment.

Since the approach to vibrational equilibrium in carbon dioxide is spread out over such a large number of collisions it is possible to observe this lag with the interferometric techniques already described. Figure 23 shows a shock of Mach number 1.134 traveling through CO_2 at 200 mm Hg pressure and 23.5°C . An initial jump in density across the shock front is followed by a

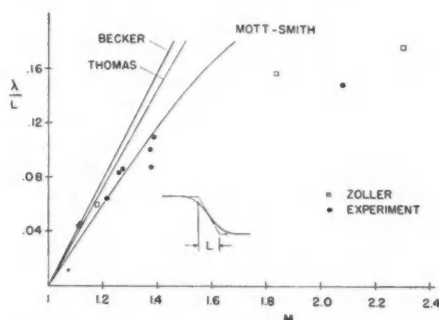


FIG. 22. Shock thickness L as a function of Mach number M . λ is Maxwell's mean free path. The circles are experimental points for argon obtained by Greene and Hornig. L is defined in the inset.

³³ K. Zoller, *Z. Physik*, **130**, 1 (1951).

³⁴ H. M. Mott-Smith, *Phys. Rev.* **82**, 885 (1951).

³⁵ G. R. Cowan and D. F. Hornig, *J. Chem. Phys.* **18**, 1008 (1950).

³⁶ E. F. Greene and D. F. Hornig, "The shape and thickness of shock fronts in argon, hydrogen, nitrogen, and oxygen," ONR Contract N7onr-358, Tech. Rep. No. 4, Brown University, 1952.

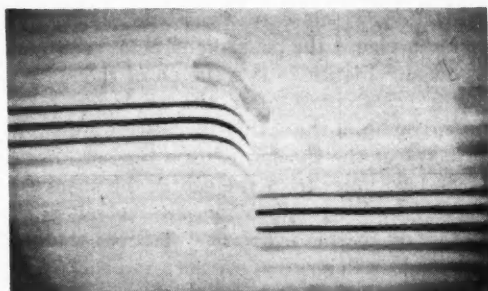


FIG. 23. A shock of Mach number 1.134 in CO₂. The region where vibrational energy gradually approaches equilibrium may be clearly seen. The two vertical hairlines are 1 in. apart. Data are given in Table II.

region of adjustment in which the density asymptotically approaches a final equilibrium value.

Bether and Teller³⁷ have given a theoretical treatment which may be used to interpret these effects. Conditions at state 2, immediately behind the shock front, may be found from Eqs. (4), (9), (10) with the assumption that only the vibrational energy remains unchanged in crossing the shock front. At 23.5°C, the specific heat at constant volume for CO₂ is divided among the various degrees of freedom as follows: $(3/2)R$ in translation, R in rotation (CO₂ is a linear symmetric molecule), and $0.95R$ in vibration. The appropriate specific heat ratio γ for calculating conditions at state 2 is therefore 1.4 and the effective Mach number is equal to the shock speed divided by the velocity of sound computed not counting the vibrational contribution to specific heat. The final equilibrium state 3 is found from Eqs. (5)–(7) taking into account the temperature dependence of enthalpy. Table II gives the values of pressure ratio, temperature, density ratio, and velocity in the three regions of Fig. 23. In their paper Bethe and Teller show that the values of velocity, density, etc., on the high-pressure side of the shock are uniquely determined by the initial conditions on the low-pressure side of the shock without reference to any of the intervening processes. It follows that the only effect of lagging heat capacity on a plane

steady shock is to increase its thickness and not to alter the final state of the gas.

If one assumes that the vibrational state may be characterized by a temperature T_{vib} then the approach to equilibrium is governed by the relation

$$dc_{vib}T_{vib}/dt = (c_{vib}(T - T_{vib})/\tau), \quad (18)$$

where c_{vib} is the vibrational specific heat and τ is the relaxation time. After a time τ the temperature difference would diminish to $1/e$, or 0.368, of its initial value in a case where the temperature change was small so that τ and c_{vib} remained constant. A fair estimate of τ may be made from Fig. 23 by measuring the distance d to the point where a given fringe has approached to within $1/e$ of its final position. The relaxation time is $\tau = d/v_2 = 11 \mu\text{sec}$. We have neglected the changing velocity and temperature as the gas flows away from the shock front and have assumed that the density also decays exponentially. Correcting for these factors makes only a small change in τ . Actually the CO₂ contains some impurities which, if eliminated, would make the relaxation time even longer.

Some experiments have been made on very strong shocks where a considerable fraction of the molecules receive enough energy to produce dissociation, electronic excitation, and ionization. A shock of Mach number 5 in argon, for example, is luminous over a region somewhat less than 100 mean free paths.¹³ Other interesting effects have been observed with cylindrically converging shocks³⁸ and shocks produced in gaseous discharge tubes.³⁹ Since the energy of a converging shock is distributed among a continually decreasing number of particles, the possibility arises of producing exceedingly high temperatures. Esti-

TABLE II. Gas properties for the shock in carbon dioxide shown in Fig. 23. Region 1 is ahead of the shock, 2 immediately behind the shock front, and 3 far enough away for complete equilibrium to exist.

Region		1	2	3
Temperature	°C	23.5	40.7	43.0
Pressure ratio	p/p_1	1	1.214	1.359
Density ratio	ρ/ρ_1	1	1.148	1.276
Flow velocity	ft/sec	1014	883	795

³⁷ H. A. Bethe and E. Teller, "Deviations from thermal equilibrium in shock waves," Reissued by University of Michigan, Engineering Research Institute, Ann Arbor, 1951.

³⁸ R. W. Perry and A. Kantrowitz, J. Appl. Phys. **22**, 878 (1951).

³⁹ Fowler, Goldstein, and Clotfelter, Phys. Rev. **82**, 879 (1951).

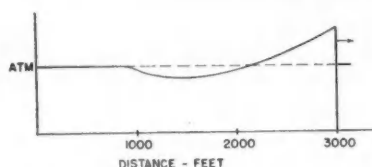


FIG. 24. Sketch of the pressure distribution 1.5 sec after the explosion of an atomic bomb. Atmospheric disturbances and the uneven ground greatly modify this idealized pattern at large distances.

mates of the temperature produced in this way have ranged well over $20\,000^{\circ}\text{K}$. Luminous shocks have been produced in a tube by discharging a large condenser across it,³⁹ and spectral analysis of the light produced showed that He^{++} , N^{++} , and N^{+++} were present in the shock. Other measurements indicate that the duration of emission is not longer than one or two μsec . Many aspects of these phenomena have not been explained.

DECAY OF SHOCKS

In previous discussions of plane shocks in tubes we have assumed that immediately behind the front the flow variables were constant and the shock proceeded without attenuation. When the rarefaction reflected from the end of the chamber catches up with the shock this state of affairs is no longer true and the shock is rapidly attenuated. Even before this happens one cannot neglect completely the growth of boundary layers on the walls and the development of resistance to the flow which serves to slowly weaken the incident shock.⁴⁰ The effect decreases with increasing tube diameter.

The shock wave generated by an explosion or other concentrated source in a homogeneous atmosphere expands in three dimensions and decays very rapidly because of the ever greater area of its surface and the irreversible processes involved. The hot gases over expand creating a

rarefaction which eats away the shock from behind giving it the peaked character commonly observed.⁴¹ Figure 24 is a sketch of the state of affairs 1.5 seconds after the explosion of an atomic bomb of the "Able" type.⁴¹ At a distance of 3000 feet the shock creates a pressure rise of 8 psi. Two miles from the center the shock has dropped to 1 psi and looking at the tail of the rarefaction in this figure one observes that the wave ends with a rise in pressure and this part is propagated into the region of the trough where the velocity of sound is lower. Evidently these are the qualitative conditions required for shock formation and, indeed, for weak shocks of short duration⁴² a stable wave form has been observed consisting of a rise in pressure through a shock followed by a linear fall to an equal pressure below atmospheric and ending in a shock returning the pressure to atmospheric again. These "*N*" waves are a logical result of the discussion of Fig. 1 for increased amplitudes. They are not observed, however, for very large explosions presumably because, at the great distances one must go from the source before the amplitude can be called small, the durations become very long and the slope of the pressure-distance curve is below the value necessary for rapid shock growth. As a matter of practice in the field, the classical shape indicated in Fig. 24 is never observed for weak blast waves at great distances since the atmosphere is far from uniform and the earth is not an ideal boundary. From thirty to several hundred miles the refractive effects of temperature gradients and wind currents lead to interfering effects from multiple paths which result in widely distorted sound waves of large amplitudes⁴³ but low fundamental frequencies of the order of one fifth cycle per second.

⁴¹ U. S. Atomic Energy Commission, *The Effects of Atomic Weapons* (McGraw-Hill Book Company, Inc., New York 1950).

⁴² DuMond, Cohen, Panofsky, and Deeds, *J. Acoust. Soc. Am.* **18**, 97 (1946).

⁴³ E. F. Cox, *J. Acoust. Soc. Am.* **19**, 832 (1947).

⁴⁰ R. J. Emrich and C. W. Curtis, *J. Appl. Phys.* **24**, 360 (1953).

Concerning new discoveries and theories, Thomas Young says, "The discovery of simple and uniform principles, by which a great number of apparently heterogeneous phenomena are reduced to coherent and universal laws, must ever be allowed to be of considerable importance toward the improvement of the human intellect."—Thomas Young, Natural Philosopher by ALEX WOOD (Cambridge University Press, 1954).

The above may be an example of Thomas Young's verbose and heavy prose by which he failed to be a clear lecturer.—ED.

Note on Perturbation Theory

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(Received February 16, 1954)

Perturbation theory is derived in such a way that the wave function renormalization terms appear automatically. The derivation is based on the Feynman theorem for the rate of change of an eigenvalue with respect to a parameter, and a corresponding theorem for the rate of change of an eigenfunction.

I. A DERIVATION OF PERTURBATION THEORY

BY means of perturbation theory one finds the eigenfunctions $\psi_j(\lambda)$ and the eigenvalues $E_j(\lambda)$ of a Hamiltonian $H(\lambda)$ as a power series in λ , where λ is a parameter. The form of these series is, of course, well known, and their derivation is an integral part of any course in quantum mechanics. However the usual derivations have the annoying feature, to this author at least, that in each order in λ the term in the expansion of $\psi_j(\lambda)$ which is proportional to $\psi_j(0)$ (the wave function renormalization term) requires special treatment.

In this article we present a derivation of perturbation theory in which no such special treatment is necessary. The method is based on the following two equations which are true for all values of λ :

$$\partial E_j / \partial \lambda = (\psi_j, \partial H / \partial \lambda \psi_j), \quad (1)$$

$$\partial \psi_j / \partial \lambda = \sum_{i \neq j} \psi_i (\psi_i, \partial H / \partial \lambda \psi_j) / (E_j - E_i). \quad (2)$$

For $\lambda=0$ these are just the usual equations of first-order perturbation theory, however, as we said, they are in fact true for all values of λ . A derivation of these equations will be given in Sec. II. We may remark here that Eq. (1) for arbitrary λ is sometimes called "Feynman's theorem" and deserves mention on its own merits in a course in quantum mechanics because of the many applications which have been made of it.¹

As an example of how Eqs. (1) and (2) allow

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¹ A partial list of references is: J. H. Van Vleck, *Phys. Rev.* **51**, 587 (1928); R. P. Feynman, *Phys. Rev.* **56**, 340 (1939); V. F. Weisskopf, *Phys. Rev.* **56**, 72 (1939); A. Pais and S. T. Epstein, *Revs. Modern Phys.* **21**, 445 (1949); J. K. Bragg and S. Golden, *Phys. Rev.* **75**, 735 (1949); T. Berlin, *J. Chem. Phys.* **19**, 208 (1951); L. L. Foldy, *Phys. Rev.* **83**, 397 (1951); S. T. Epstein, *Progr. Theoret. Phys. (Japan)* **6**, 441 (1951).

one to derive the formulas of higher-order perturbation theory in a completely automatic fashion, let us work out the second-order formulas. For this we must compute $\partial^2 E_j / \partial \lambda^2$ and $\partial^2 \psi_j / \partial \lambda^2$. Turning first to $\partial^2 E_j / \partial \lambda^2$ we have, by differentiating Eq. (1) and using primes to indicate differentiation with respect to λ

$$E_j'' = (\psi_j', H' \psi_j) + (\psi_j, H' \psi_j') + (\psi_j, H'' \psi_j).$$

If we now substitute from Eq. (2) and introduce the notation $\omega_{ji} \equiv E_j - E_i$ and $h_{ij} \equiv (\psi_i, H' \psi_j)$ then we find

$$E_j'' = 2 \sum_{i \neq j} \frac{h_{ji} h_{ij}}{\omega_{ji}} + (\psi_j, H'' \psi_j), \quad (3)$$

which, for $\lambda=0$, is just the usual second-order result. To compute ψ_j'' we first differentiate Eq. (2) to obtain

$$\begin{aligned} \psi_j'' = & \sum_{k \neq j} \psi_k' \frac{(\psi_k, H' \psi_j)}{\omega_{jk}} \\ & + \sum_{i \neq j} \psi_i \frac{(\psi_i', H' \psi_j)}{\omega_{ji}} + \sum_{i \neq j} \psi_i \frac{(\psi_i, H' \psi_j')}{\omega_{ji}} \\ & - \sum_{i \neq j} \psi_i \frac{(\psi_i, H' \psi_j)}{(\omega_{ji})} (E_j' - E_i') \\ & + \sum_{i \neq j} \psi_i \frac{(\psi_i, H'' \psi_j)}{\omega_{ji}}, \end{aligned}$$

which becomes, on substitution from Eqs. (1) and (2)

$$\begin{aligned} \psi_j'' = & \sum_{k \neq j} \sum_{i \neq k} \psi_i \frac{h_{ik} h_{kj}}{\omega_{ki} \omega_{jk}} + \sum_{i \neq j} \sum_{k \neq i} \psi_i \frac{h_{ik} h_{kj}}{\omega_{ki} \omega_{ji}} \\ & + \sum_{i \neq j} \sum_{k \neq j} \psi_i \frac{h_{ik} h_{kj}}{\omega_{ji} \omega_{jk}} + \sum_{i \neq j} \psi_i \frac{h_{ij} h_{ii}}{(\omega_{ji})^2} \\ & - \sum_{i \neq j} \psi_i \frac{h_{ij} h_{ij}}{(\omega_{ji})^2} + \sum_{i \neq j} \psi_i \frac{(\psi_i, H'' \psi_j)}{\omega_{ji}}. \quad (4) \end{aligned}$$

Finally, by a little obvious rearranging of the first two sums and the use of the identity $(\omega_{ki}\omega_{jk})^{-1} + (\omega_{ik}\omega_{ji})^{-1} = (\omega_{jk}\omega_{ji})^{-1}$, Eq. (4) can be put into the standard second-order form including the renormalization term (which arises from the first sum), namely

$$\begin{aligned}\psi_j'' = & -\psi_j \sum_{k \neq j} \frac{h_{jk}h_{kj}}{(\omega_{kj})^2} \\ & + 2 \sum_i \sum_{k \neq j} \psi_i \frac{h_{ik}h_{kj}}{\omega_{ji}\omega_{jk}} - 2 \sum_{i \neq j} \psi_i \frac{h_{ij}h_{ji}}{(\omega_{ji})^2} \\ & + \sum_{i \neq j} \psi_i \frac{(\psi_i, H'' \psi_j)}{\omega_{ji}}.\end{aligned}$$

II. DERIVATION OF EQUATIONS (1) AND (2)

Clearly one derives Eqs. (1) and (2) by calculating the first-order changes in ψ_j and E_j brought about by changing λ to $\lambda + \delta\lambda$. As a variation on the usual first-order perturbation theory technique [the only change in the standard treatment would be to replace $\psi_j(0)$ by $\psi_j(\lambda)$ everywhere] we will proceed as follows.² We start from $E_j\delta_{ij} = (\psi_i, H\psi_j)$ and differentiate to find

$$E_j'\delta_{ij} = (\psi_i', H\psi_j) + (\psi_i, H\psi_j') + (\psi_i, H'\psi_j),$$

which, using the hermiticity of H and the equations $H\psi_i = E_i\psi_i$ and $H\psi_j = E_j\psi_j$, becomes

$$E_j'\delta_{ij} = E_j(\psi_i', \psi_j) + E_i(\psi_i, \psi_j') + (\psi_i, H'\psi_j).$$

But we also have

$$(\psi_i', \psi_j) + (\psi_i, \psi_j') = (\psi_i, \psi_j)' = \delta_{ij}' = 0, \quad (5)$$

² Essentially the same derivation is given, for example, by Pauli in his *Die Allgemeinen Prinzipien der Wellenmechanik* (J. W. Edwards, Ann Arbor) on page 162 as part of his discussion of adiabatic and sudden processes. We reproduce the proof here for completeness.

so we can write

$$E_j'\delta_{ij} = (E_i - E_j)(\psi_i, \psi_j') + (\psi_i, H'\psi_j).$$

This is our fundamental equation. For $i = j$ it is just Eq. (1), while for $i \neq j$ we find

$$(\psi_i, \psi_j') = (\psi_i, H'\psi_j) / (E_j - E_i),$$

which would be equivalent to Eq. (2) if (ψ_j, ψ_j') , which we shall denote by α_j , were equal to zero. We shall now complete the derivation of Eqs. (1) and (2) by showing that one, in fact, can choose $\alpha_j = 0$.³ To do this we remark first of all that from Eq. (5) we have $\alpha_j + \alpha_j^* = 0$, i.e., α_j is pure imaginary. We will now show that the value of α_j is physically irrelevant, and that in particular α_j may be taken equal to zero, by showing that one can change the value of α_j by changing the phase of ψ_j by arbitrary constant amounts. To see this write $\psi_j = e^{i\phi_j}\tilde{\psi}_j$, where ϕ_j is a real function which depends only on λ and hence is physically irrelevant. A short calculation now shows that

$$\alpha_j = i\phi_j' + \beta_j,$$

where $\beta_j = (\tilde{\psi}_j, \tilde{\psi}_j')$. Note that this is a consistent equation since both α_j and β_j are purely imaginary and functions only of λ . Thus we see that for a given $\tilde{\psi}_j$ we may, by varying ϕ_j , give α_j any value we want, and, in particular, the value zero. As a special case of the physical irrelevance of α_j we may remark that had we not chosen $\alpha_j = 0$, then instead of Eq. (3) we would have

$$E_j'' = (3) + (\alpha_j + \alpha_j^*)(\psi_j, H'\psi_j);$$

but we have seen that $\alpha_j + \alpha_j^* = 0$, so that the energy is unaffected by the value of α_j .

³ It is true that the evaluation of α_j requires a special treatment. However, it is a final treatment. Once we find the value of α_j , then the rest of perturbation theory follows automatically. One does not meet a new problem in each order.

Applied Mathematics

The Society for Industrial and Applied Mathematics will hold its first national meeting in conjunction with the annual meetings of the American Mathematical Society, the Mathematical Association of America, and the Association for Symbolic Logic at the University of Pittsburgh, on December 27-29. The following addresses will be presented to an evening meeting: "The History of a Problem," Dr. Brockway McMillan, *Bell Telephone*

Laboratories; "The Control of Industrial Operations," Professor Herbert A. Simon, *Carnegie Institute of Technology*; "Probability Theory in Liability and Property Insurance," Mr. C. W. Crouse, *Actuary, Preslan and Company*. Further information can be obtained from H. W. Kuhn, Dalton Hall, Bryn Mawr College, Bryn Mawr, Pennsylvania.

A Way of Representing the Motion of a Classical System as Being along a Geodesic in Space-Time

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(Received March 10, 1954)

Ordinarily, the Jacobi formulation of classical mechanics describes a system by giving the path of a "system point" in "configuration space," the path being a geodesic if a metric is suitably defined. Nothing is said, however, about the motion in time.

It is suggested here that, by adding a degree of freedom in uniform motion to serve as a clock, and identifying the time with the coordinate of that degree of freedom, the configuration space can be extended to a space-time, in which the motion is completely described, both in space and in time, as being along a geodesic. Also, by this representation, the idea of "absolute time" is made unnecessary.

A representation such as this is, of course, normal in relativistic theory, but its use here in nonrelativistic mechanics is believed to have some novelty.

THE Jacobi form of the principle of least action is ordinarily used to express the motion of a system by giving a curve in the "configuration space" of the system such that successive configurations of the system are represented by successive points on the curve.^{1,2} With a proper choice of metric, this curve becomes a geodesic in the configuration space. This representation, however, gives no information about the motion in time. While it would be interesting to have a "configuration space-time" with a suitable metric, such that the motion of the system would be along a geodesic in this space time, this is ordinarily accomplished only approximately or not at all in classical mechanics.³

This discussion proposes that the desired space-time representation can be obtained by including a suitable "clock" as part of the system. The clock might be, say, a sphere rotating uniformly about an axis. This sphere has then a degree of freedom represented by θ , its angular position. The view will be taken that "the time" is merely a quantity proportional to θ . Since θ gives the system an extra degree of freedom, this degree of freedom can be applied in the Jacobi theory, and then interpreted as time.

The principle of least action may be expressed

in the form that

$$\delta \int d\sigma = 0, \quad (1)$$

i.e., that the integral is an extremum, for the actual path. Here $d\sigma$ is an "element of arc," given by

$$(d\sigma)^2 = \sum_i \sum_j \{E - V(q)\} m_{ij} dq_i dq_j, \quad (2)$$

the q 's being the coordinates; E the total energy; $V(q)$ the potential energy, a function of some or all of the q 's; and m_{ij} the elements of the metric tensor, being such that the kinetic energy T is given by

$$T = \frac{1}{2} \sum_i \sum_j m_{ij} \left(\frac{dq_i}{dt} \right) \left(\frac{dq_j}{dt} \right). \quad (3)$$

Here the coordinate θ , proportional to time, will just be considered along with the others. In fact, by supposing that the sphere used as a clock makes $1/(2\pi)$ revolutions per second, θ can be set equal to time. Thus the actual motion is described in time.

For a first example, consider the motion of a small sphere, of mass m , acted on by no forces, and acting as its own clock by revolving about an axis at $1/(2\pi)$ revolutions per second. The coordinates will be written

$$\text{also } \left. \begin{aligned} q_1 &= x, & q_2 &= y, & q_3 &= z, & q_4 &= \theta; \\ m_{11} &= m_{22} = m_{33} = m \cdot m_{44} &= I \end{aligned} \right\} \quad (4)$$

¹ H. Goldstein, *Classical Mechanics* (Addison-Wesley, Cambridge, 1950), Sec. 7-5.

² L. Brillouin, *Les Tenseurs en Mécanique et en Élasticité* (Masson et Cie, Paris, 1949), Chap. 8, Secs. 5 and 6.

³ See reference 2, Sec. 7.

and all other m 's are zero. The moment of inertia i.e., of the sphere about its axis is I .

Then Eq. (1) gives

$$\delta \int [m\{(dx)^2 + (dy)^2 + (dz)^2\} + I(d\theta)^2]^{\frac{1}{2}} = 0. \quad (5)$$

To make an integral

$$\int \phi(x, y, z, \theta, dx, dy, dz, d\theta) \quad (\phi \text{ any function})$$

an extremum, the solution is⁴

$$\frac{\partial \phi}{\partial x} - d \left\{ \frac{\partial \phi}{\partial (dx)} \right\} = 0 \quad (6)$$

and similarly for the other variables. When this is applied to Eq. (5) one finds

$$d \left[\frac{m dx}{[m\{(dx)^2 + (dy)^2 + (dz)^2\} + I(d\theta)^2]^{\frac{1}{2}}} \right] = 0, \quad (7)$$

$$\delta \int [a/r - E] \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\}^{\frac{1}{2}} = 0. \quad (9)$$

Then from Eq. (6)

$$d \left[\frac{(a/r - E)mr^2 d\psi}{[a/r - E] \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\}^{\frac{1}{2}}} \right] = 0, \quad (10)$$

or

$$\begin{aligned} \frac{(a/r - E)mr^2 d\psi}{[a/r - E] \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\}^{\frac{1}{2}}} &= \text{constant} = k_2, \\ \frac{(a/r - E)Id\theta}{[a/r - E] \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\}^{\frac{1}{2}}} &= \text{constant} = k_3, \end{aligned} \quad (11)$$

and

$$\begin{aligned} \frac{-a/2r^2 \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\} + \{a/r - E\}mr(d\psi)^2}{[a/r - E] \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\}^{\frac{1}{2}}} \\ - d \left[\frac{(a/r - E)m dr}{[a/r - E] \{m(dr)^2 + mr^2(d\psi)^2 + I(d\theta)^2\}^{\frac{1}{2}}} \right] = 0. \end{aligned} \quad (12)$$

Equations (10) and (11) give

$$\frac{mr^2 d\psi}{I d\theta} = \frac{k_2}{k_3} = \text{constant}, \quad (13)$$

which is just Kepler's second law. Equations (10)

⁴F. S. Woods, *Advanced Calculus* (Ginn and Company, New York, 1934), Chap. 14, Sec. 129.

$$\frac{dx}{[m\{(dx)^2 + (dy)^2 + (dz)^2\} + I(d\theta)^2]^{\frac{1}{2}}} = k_1 = \text{constant}$$

and similarly for the other variables. Thus this result amounts to

$$dx:dy:dz:d\theta = k_1:k_2:k_3:k_4, \quad (8)$$

the k 's being constants; i.e., motion is in a straight line, and uniform in time.

As a second example, consider the motion of a sphere in a central inverse-square force field, the sphere being of mass m and moment of inertia about its axis I . Besides the θ coordinate, the radial and angular position coordinates r and ψ will determine the configuration. Again, by having the sphere make $1/(2\pi)$ revolutions per second, θ will eventually be identified with the time. Then, $q_1 = r$, $q_2 = \psi$, $q_3 = \theta$, $m_{11} = m$, $m_{22} = mr^2$, $m_{33} = I$; the other m 's are zero. $V(q) = -a/r$, and E may be negative. Then Eq. (1) becomes

and (12) give

$$\frac{k_2 d\psi}{r} - \frac{a}{2r^2} \frac{mr^2 d\psi}{k_2} = d \left\{ \frac{k_2 dr}{r^2 d\psi} \right\} = -d \left\{ \frac{d}{d\psi} \left(\frac{k_2}{r} \right) \right\} \quad (14)$$

or

$$\frac{d^2}{d\psi^2} \left(\frac{k_2}{r} \right) + \frac{k_2}{r} - \frac{am}{2k_2} = 0 \quad (15)$$

and a solution is

$$r = \frac{2k_2^2}{am + 2Ak_2 \cos(\psi - \psi_0)}, \quad (16)$$

A and ψ_0 being constants of integration. Equation (16) is just the equation of a conic, which is the orbit of the moving body. Thus a complete description of the motion can be obtained from Eq. (9) along with the initial conditions, and it is expressed by Eqs. (13) and (16).

These examples demonstrate how, by using uniform motion of an actual body as a "clock," the motion of a mechanical system may be described as proceeding along a geodesic in a configuration space-time with a suitable metric.

APPENDIX

The introduction of an "ignorable coordinate" (θ in the examples), proportional to time, could also be considered as providing a short cut between the principles of least action and Hamilton's principle. Equations (1), (2), and (3) state the principle of least action as

$$\delta \int [E - V(q)] \sum_i \sum_j m_{ij} dq_i dq_j = 0. \quad (17)$$

The coordinate θ is included among the q 's; let it be q_a . Since there is no interaction between θ and the other coordinates, Eq. (17) may be written

$$\delta \int [E - V(q)] \{ m_{aa} + \sum_{i \neq a} \sum_{j \neq a} (dq_i/d\theta) \times (dq_j/d\theta) \} d\theta = 0, \quad (18)$$

where m_{aa} is the m parameter of the clock introduced into the discussion; in the examples it has been I , the moment of inertia of the sphere. The above amounts to:

$$\delta \int [(E - V)T] d\theta = 0 \quad (19)$$

and the Euler-Lagrange equations for this extremum problem are

$$\left\{ \frac{E - V}{T} \right\} \left[\frac{d}{d\theta} \left\{ \frac{\partial T}{\partial (dq_i/d\theta)} \right\} - \frac{\partial T}{\partial q_i} \right] + \left\{ \frac{T}{E - V} \right\} \frac{\partial V}{\partial q_i} = 0 \quad (20)$$

for every i . Since $E - V = T$, and V does not involve $dq/d\theta$, this is just

$$\frac{d}{d\theta} \frac{\partial (T - V)}{\partial (dq_i/d\theta)} - \frac{\partial (T - V)}{\partial q_i} = 0 \quad (21)$$

for every i . Since θ is identified with time, these are just Lagrange's equations. It is interesting that m_{aa} has disappeared completely. Equation (21) is the Euler-Lagrange equation for the extremum problem:

$$\delta \int (T - V) dt = 0, \quad (22)$$

i.e., for Hamilton's principle, which is thus obtained from the principle of least action.

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Phase, Group, and Signal Velocity

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(Received March 8, 1954)

Phase, group, and signal velocity are defined. Simple examples are given in which the phase and group velocities exceed the velocity of light, in which the phase velocity is a function of distance, and in which the group velocity is equal to the signal velocity.

INTRODUCTION

THE phase velocity associated with wave phenomena is the velocity which an observer at any point would need in order to detect no changes of phase in an advancing sinusoidal wave. In this discussion we consider only the propagation of light. It is well known that in many instances the phase velocity exceeds the velocity of light in free space. According to relativity, it is not possible to send signals with a velocity faster than the velocity of light because signaling requires transmission of energy, and this cannot occur with a speed greater than that of light. For some purposes it is convenient to introduce a quantity called the *group velocity*. The group velocity is the velocity which an observer would need in order to detect no change in the amplitude of an amplitude-modulated wave. The quantity which determines the actual speed of signaling is called the *signal velocity*. This discussion is intended to clarify the meanings of these three quantities by means of some very simple examples. This problem was discussed by Brillouin¹ and by Sommerfeld.² Their treatment is thorough and suitable for advanced students. Very few elementary discussions are available.

EXAMPLES

First we consider the phase velocity. If the wavelength is known to be λ and the frequency is ν , then ν waves pass an observer each second, and if the observer moves with the waves with velocity $v_p = \nu\lambda$, he will stay with a particular wave and see no change in phase. The phase velocity is, therefore, ordinarily equal to $\nu\lambda$. Consider the following example. An observer is

confined to a long straight trench (see Fig. 1). Electromagnetic radiation is incident on the trench from the direction shown. Successive wave fronts are drawn. If the observer measures the wavelength, he does so along the direction of the trench. His measured wavelength is $\lambda' = oo'$. If λ is the free-space wavelength, the wavelength measured by him is $\lambda' = \lambda/\cos\theta$. The phase velocity is $\nu\lambda' = \nu\lambda/\cos\theta = c/\cos\theta$, where c is the velocity of light in free space. The observer must move in the direction of the trench with velocity equal to $c/\cos\theta$ if he is to detect no change in phase. The phase velocity is seen to be faster than c .

Suppose that instead of monochromatic radiation we employ a modulated wave. This could be accomplished by using two monochromatic waves, close together in frequency. The electric field vector could be described by the relation

$$E = E_1 [\cos(\omega t - \beta z) + \cos[(\omega + \Delta\omega)t - (\beta + \Delta\beta)z]], \quad (1)$$

where $\beta = \omega/v_p$; we can use a trigonometric identity to transform Eq. (1) to

$$E = 2E_1 \cos \frac{1}{2} [(2\omega + \Delta\omega)t - (2\beta + \Delta\beta)z] \times \cos \frac{1}{2} [\Delta\omega t - \Delta\beta z]. \quad (2)$$

If $\Delta\omega$ and $\Delta\beta$ are small, we can write

$$E \approx 2E_1 \cos \left(\frac{\Delta\omega}{2} t - \frac{\Delta\beta}{2} z \right) \cos(\omega t - \beta z). \quad (2A)$$

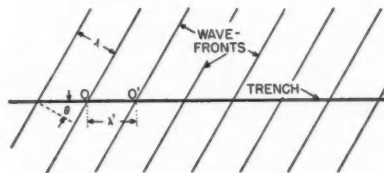


FIG. 1. Wave fronts obliquely incident on a trench.

¹ L. Brillouin, *Congres International d'Electricite*, Vol. II, 1^{re} Sec., Paris, 1932.

² A. Sommerfeld, *Ann. Physik* **44**, 177-202 (1914).

The first factor in Eq. (2A) represents the propagation of the modulation envelope. We can use Eq. (2A) to discuss group velocity, by inquiring what velocity an observer would need in order to observe no changes in amplitude. This velocity is obtained by setting the factor $\cos(\frac{1}{2}\Delta\omega t - \frac{1}{2}\Delta\beta z) = \text{constant}$; this gives

$$z = \frac{\Delta\omega}{\Delta\beta} t = v_g t, \quad (3)$$

$$v_g = \frac{\Delta\omega}{\Delta\beta}. \quad (4)$$

In general the angular frequency is a function of the wavelength, and from Eq. (4) we define the group velocity as $v_g = d\omega/d\beta$. For the case of Fig. 1, we have for the phase velocity

$$v_p = v\lambda' = \frac{2\pi\nu}{\beta} = \frac{\omega}{\beta} = \frac{c}{\cos\theta}. \quad (5)$$

We can obtain the group velocity by getting $d\omega/d\beta$ from Eq. (5), obtaining

$$v_g = \frac{d\omega}{d\beta} = \frac{c}{\cos\theta}. \quad (6)$$

We see that the group velocity as given by expression (6) is again greater than the velocity of light. It is easy to see directly why this should be so if we imagine the wave to be amplitude-modulated by increasing the amplitude of every tenth wave, as shown in Fig. 2. If the observer is to see no change in amplitude, and he is initially at point A, he needs to move along the trench with velocity $v_g = c/\cos\theta$.

We next inquire what is the maximum speed with which a signal can be sent from point A to point B (see Fig. 2). In order for the observer

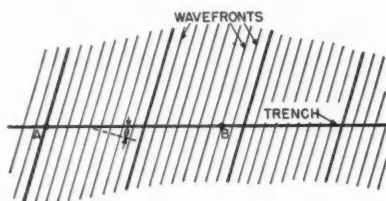


FIG. 2. Wave fronts with every tenth wave of large amplitude obliquely incident on a trench.

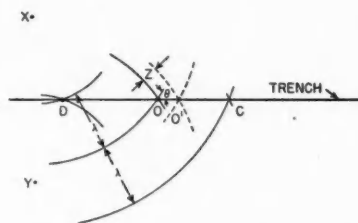


FIG. 3. Trench illuminated by two sources.

to signal he must do more than watch the light waves go by. He must originate some sort of disturbance. One way would be to interact with the waves going by at A and modify them in some way. If he does this, spherical wavelets will emerge from A, according to Huygens' principle. These wavelets travel from A to B with the velocity of light c . Thus we see that the maximum speed of signaling is c , but the phase and group velocities both exceed c .

We consider another example. Suppose the trench is illuminated by two sources at X and Y (Fig. 3). At point O the wavelength is OC if measured to the right and OD if measured to the left. We see that the wavelength in the trench direction depends on the distance and on the direction in which we measure, and, in general, is not the same at any two points. We can calculate the phase velocity at O as follows. We imagine another wave front (shown dotted) to be very slightly ahead of the solid one through O, in phase. The time required for the wave to go from O to O' is $\Delta t = z/c$. In this time the observer would have to move from O to O', which is $z/\cos\theta$. The phase velocity in this case is, therefore,

$$v_p = \frac{\frac{z}{\cos\theta}}{\frac{z}{c}} = \frac{c}{\cos\theta}. \quad (7)$$

We can write $c = v\lambda = \omega\lambda/2\pi$; utilizing this in Eq. (7), we obtain

$$v_p = \frac{\omega\lambda}{2\pi \cos\theta} = \frac{\omega}{\beta}. \quad (7A)$$

The group velocity can be obtained by evalu-

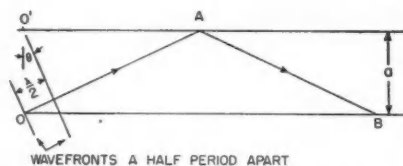


FIG. 4. Plane waves in a rectangular wave guide.

ating $d\omega/d\beta$, from Eq. (7A). This gives

$$v_g = \frac{d\omega}{d\beta} = v_p = \frac{c}{\cos\theta}. \quad (8)$$

The group velocity is also $c/\cos\theta$. The signal velocity is again determined to be c because waves originated at some point along the trench will be propagated with velocity c .

As a final example we consider propagation of electromagnetic waves in a rectangular wave guide, see Fig. 4. The radiation inside of the wave guide can be considered to be made up of plane waves reflected back and forth from the walls. The signal velocity is seen to be the distance OB divided by the time required for light to go from O to B along the path OAB .

$$\text{Signal velocity} = \frac{OB}{\frac{OA+AB}{c}} = c \cos\theta. \quad (9)$$

We consider the propagation mode which has the lowest cut-off frequency. For this mode the points O and O' of the wave system must be on wave fronts a half-period apart, to satisfy the boundary conditions. This requires

$$\sin\theta = \frac{\lambda}{2a}, \quad \text{and} \quad \cos\theta = \left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}.$$

Employing these relations in Eq. (9), we obtain

$$\text{Signal velocity} = c \left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}. \quad (10)$$

Let λ' be the wavelength in the direction OB . Then

$$\lambda' = \frac{\lambda}{\cos\theta}, \quad (11)$$

$$\lambda' = \frac{\lambda}{\left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}}, \quad (12)$$

$$\beta = \frac{2\pi}{\lambda'} = \frac{2\pi \left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}}{\lambda}, \quad (13)$$

$$c = \frac{\omega\lambda}{2\pi}. \quad (14)$$

The group velocity is $d\omega/d\beta$ and can be calculated using Eqs. (12), (13), and (14). The result is

$$v_g = c \left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}.$$

The group velocity is seen to be equal to the signal velocity, and both are given by Eq. (10). The phase velocity is $v\lambda'$ where λ' is given by Eq. (12), and is

$$v_p = v\lambda' = \frac{v\lambda}{\left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}} = \frac{c}{\left\{ 1 - \left(\frac{\lambda}{2a} \right)^2 \right\}^{\frac{1}{2}}},$$

which is greater than the velocity of light.

In a dispersive medium, the phase and group velocities are sometimes less than c and sometimes greater than c . When a signal is sent, the first impulse always arrives with velocity c ; then the amplitude builds up in a complex way as described by Sommerfeld and Brillouin. The time of detection of the signal depends on the sensitivity of the detector.

Paper Representations of the Noncubic Crystal Classes

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(Received January 25, 1954)

It is shown that the full symmetries of twenty-seven of the thirty-two crystal classes, all except the five cubic classes, can be exhibited by suitably marked pieces of paper or cardboard. It is necessary to use both sides of the paper, and to exploit its small but finite thickness.

THE writer recently noticed that the full symmetries of twenty-seven of the thirty-two crystal classes, all except the five cubic classes, can be exhibited by suitably marked pieces of paper or cardboard. This does not mean that the representations are two-dimensional, for both sides of the paper are used. It is essen-

tial to assume, in some cases, that an axis or a plane of symmetry can be passed midway between the two surfaces of the paper. Thus the small but finite thickness of the paper is exploited.

Each polygon in the accompanying figures should be thought of as a paper or cardboard cutout, with the triangular areas between the solid lines and the two intersecting edges blacked in on the upper surface. The triangles bounded by the dotted lines and the two intersecting

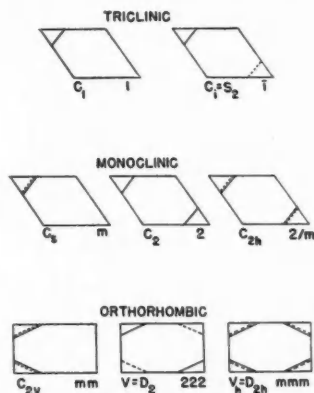


FIG. 1. The two triclinic, three monoclinic, and three orthorhombic classes.

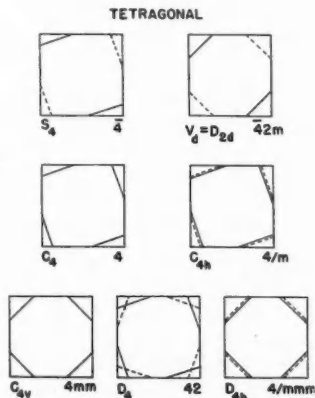


FIG. 2. The seven tetragonal classes.

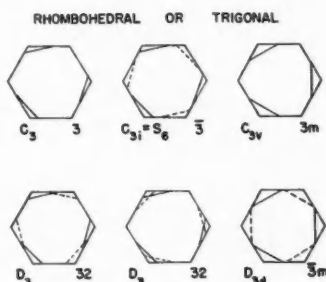


FIG. 3. The five rhombohedral or trigonal classes; two representations are given for the class D_3 or 32 (see text).

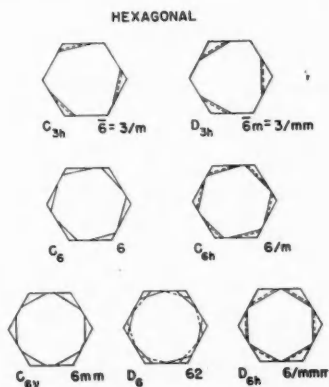


FIG. 4. The seven hexagonal classes.

edges are to be blacked in on the lower surface. Where the upper and lower blacked areas coincide, the solid and dotted lines are drawn slightly displaced from each other. Both the Schoenflies and the Hermann-Mauguin notations are given for each class. Two representations are given for the rhombohedral or trigonal class D_3 or 32 (Fig. 3); in the first the twofold axes are

perpendicular to the edges of the hexagon, and in the second they pass through the corners. Needless to say, the forms used in Figs. 1-4 are not unique.

This material is presented here in the belief that such easily made, and compact representations will prove to be of value in the study of crystallography, especially in the classroom.

Teaching Transistor Physics*

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(Received April 13, 1954)

In June, 1952, the Bell Telephone Laboratories, Murray Hill, New Jersey, sponsored a summer school for teachers of transistor physics. During the sessions the basic ideas of semiconductors were studied, and a series of fundamental experiments was performed. At Carnegie Institute of Technology the student interest in these experiments has been very high and the results were so gratifying that we felt it would be most welcome if the Bell Telephone Laboratories made these experiments available to colleges which were not represented at the transistor sessions. This they have kindly agreed to do. A brief discussion of the experiments developed by the Bell Telephone Laboratories is given in this article.

SINCE its discovery, about six years ago, the transistor has rapidly developed from a laboratory wonder to its present useful state. Today transistors of many types are in mass production, new varieties are being developed, and the potential uses for them are increasing rapidly.

If the transistor industry follows the growth pattern that practically all industries in the past have shown, then it is apparent that a large number of people will have to be trained in transistor physics. One of the major purposes of the school is to provide the basic training that industry needs. In the case of transistor physics this means that schools should offer more courses in solid-state physics with particular emphasis on semiconductors. Special courses in transistors should also be made available to those students desiring to specialize in this field as well as to practicing engineers who would like to become sufficiently well acquainted with transistors to take advantage of their capabilities.

Another duty of the school is to guide students into those fields in which they will be happy and are likely to find steady employment. A growing industry generally offers more security than one which is near its maximum growth. Figure 1 shows a growth curve similar to one that the transistor industry might be expected to follow. Almost all industries have followed the same pattern. This curve, which gives the total number of kilowatt hours of electrical energy developed each year as a function of the time, is typical of practically all industries. Growth curves are generally drawn on a semilog scale since equal vertical deviations represent equal percentage changes, thus making it easy to extrapolate the curve into the future. The positions of various industries in their growth patterns are shown in Fig. 2. Since the transistor industry would lie on the steep part of the curve near the origin, it is evident that the number of people that could be profitably employed in it will increase very rapidly for a large number of years. Normal business fluctuations will not be felt in it for a long time. For instance, during a period of business recession an industry that

* This article is based on a paper presented by the author at the June, 1953, meeting of the American Association of Physics Teachers held in Pittsburgh, Pennsylvania.

has reached its maximum growth might have to reduce the number of employees by, say, a thousand over a period of one year. However, a growing industry of comparable size, according to its normal growth curve, might call for an increase of 1100 employees in this same length of time. Therefore, instead of having to reduce their personnel by a thousand because of the business recession, they would still be able to increase their number by 100. Thus, growing industries are valuable to our economic welfare.

The transistor industry is also valuable to our national security. The small size, low power requirement, and the ruggedness of the transistor have already improved the communications systems upon which our national defense is dependent. Let schools not be a deterrent in its normal development.

Although transistors have been studied for some time, it has only been relatively recently that most of the difficulties of teaching transistor physics have been eliminated. The first difficulty is naturally concerned with having sufficient fundamental understanding of transistors to make the teaching of them worthwhile. The theory of the point-contact transistor, which was the first type of transistor to be discovered, remained in doubt for some time. Even at the present time there is some uncertainty concerning it. Upon the discovery of the junction transistor by W. Shockley, however, the theory of the point-contact transistor was aided materially.

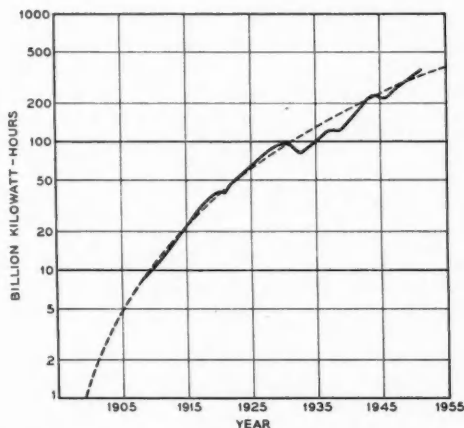


FIG. 1. Electrical energy production in the United States.

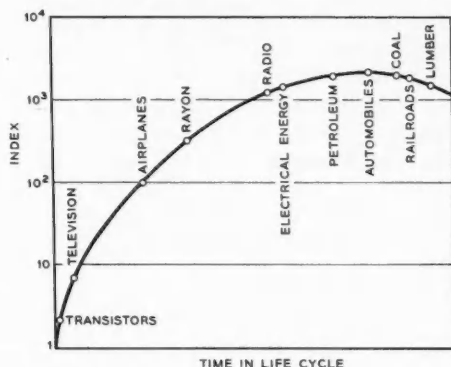


FIG. 2. Position of various U. S. industries in their growth curve (after an idea by Babson).

The theory of the junction transistor is sound and can be understood without much difficulty. The ideas involved in the junction transistor, and the experiments that are possible with it, are of considerable value in understanding the nature of solids in general.

The second early difficulty in teaching transistor physics was the lack of a suitable textbook which would enable a person not actively engaged in research in the field to gain knowledge of the subject without extensive literature surveys and study of numerous individual papers. This difficulty was overcome upon publication of Shockley's book *Electrons and Holes in Semiconductors*.¹ This textbook is particularly well suited for teaching since it is divided into three parts of increasing difficulty, thus making it a text which can be used at various levels.

A third difficulty in teaching transistor physics concerned laboratory work. Before a student makes a study of vacuum tubes he normally first studies some of the fundamental principles of electricity such as the motion of electrons in a vacuum under the influence of an electric field. In the same way, if one wishes to understand transistors, it is necessary to study the motions of electrons and holes in semiconductors, and, in particular, to understand such concepts as mobility, lifetime, and diffusion of electrons and holes. Laboratory experiments, as in studying all physics, are valuable in clarifying difficult ideas. However, in order to do fundamental ex-

¹ W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950).

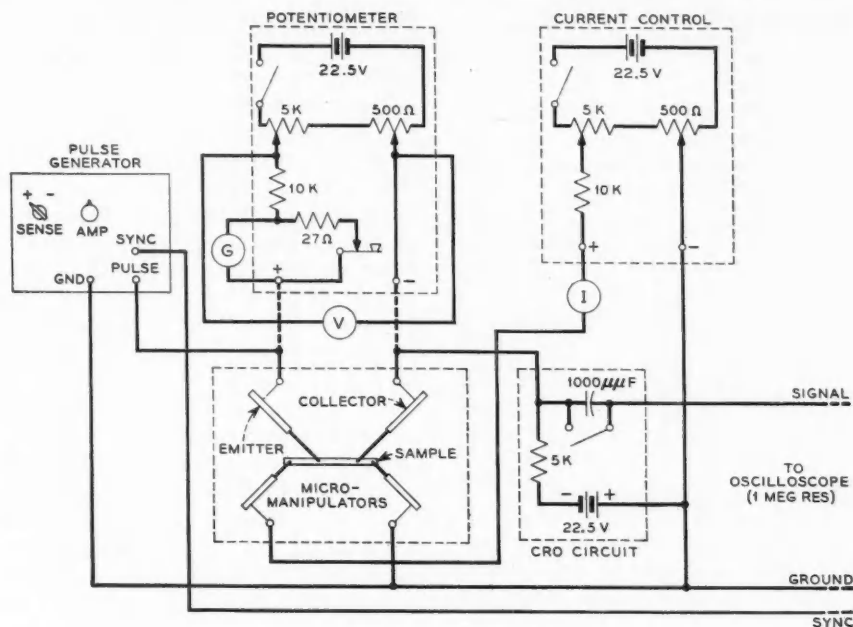


FIG. 4. Drift mobility experiment circuit.

report. It can be seen that the instructions are in sufficient detail to enable a school to set up the experiments without excessive labor.

Purpose.—To measure the drift mobility of injected holes and electrons in germanium and to give reality to the concept of hole and electron flow in solids.

References to Literature.—(1) "Hole Injection in Germanium—Quantitative Studies and Filamentary Transistors," Shockley, Pearson, and Haynes, *Bell System Tech. J.* **28**, 344 (1949).

(2) "The Mobility and Life of Injected Holes and Electrons in Germanium," J. R. Haynes and W. Shockley, *Phys. Rev.* **81**, 835 (1951).

(3) "The Mobility of Electrons in Silver Chloride," J. R. Haynes and W. Shockley, *Phys. Rev.* **82**, 935 (1951).

(4) "The Drift Mobility of Electrons in Silicon," J. R. Haynes and W. C. Westphal, *Phys. Rev.* **85**, 680 (1952).

Discussion of Experiment.—As a result of new experimental techniques developed in connection with the transistor program, the speed with which holes and electrons drift in electric fields in germanium can be measured with an accuracy

sufficient to determine their mobilities to within a few percent. The basic phenomena which permit these new experiments to be performed are those of carrier injection and collector action; with the aid of the former, holes or electrons can be injected at a given place and time, and with the aid of the latter their arrival time at another point can be determined.

It should be emphasized that transit-time measurements determine drift mobility in a very direct way. Hall-effect measurements, which have previously been used to determine mobilities in semiconductors, are quite indirect. The relationship between Hall mobility, defined by the angle between current and electric vectors in unit magnetic field, and true mobility involves the detailed nature of the scattering processes and energy-band shapes and only under special conditions does the ratio have the conventional value of $3\mu/8$.

The essence of the technique described is incorporated in the circuit shown in Fig. 4. A rod of *n*-type germanium, cut from a single crystal, is provided with rhodium-plated contacts at either end for good electrical contact. The

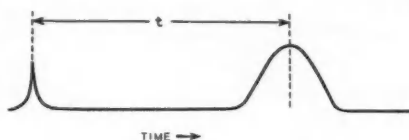


FIG. 5. Oscilloscope trace obtained in drift mobility experiment.

sample is placed on an insulating base equipped with four micromanipulators each of which controls a phosphor bronze point. Two of the points are placed in contact with the end platings of the sample. The other two are in contact with the germanium crystal and serve as emitter and collector. A variable potential source (current control circuit) is placed in series with the plated contacts so that a direct current I flows in the crystal from left to right producing an electric field in the same direction. One end of the crystal is connected to the ground. A pulse generator which produces a positive pulse having a duration of 0.5 microsecond and a repetition rate of 60 pulses per second is placed in series with the emitter point and ground. During the time of the pulse, a current flows from the emitter point into the crystal making this point an emitter of positive holes. In series with the collector point and ground is placed a resistance and a battery (C. R. O. circuit) so that this point is biased negatively as in a type A transistor. The input amplifier of a cathode-ray oscilloscope is connected across the resistance. This resistance has a value which is small compared with the impedance of the collector point so that the voltage applied to the oscilloscope is proportional to the admittance of the point. At a certain instant of time a positive pulse of current is applied to the emitter point. Holes enter the crystal at this time and are swept down to the collector in the electric field produced by the direct current I . When they arrive at the collector they produce a measurable signal in the oscilloscope.

A drawing of the trace obtained on the oscilloscope is shown in Fig. 5. The oscilloscope is synchronized to the start of the injection pulse. This pulse is visible on the oscilloscope trace because the momentarily increased current in the germanium rod serves to produce an analogous change in the potential of the crystal at the collector contact. After this initial pulse, the signal

remains constant. During this time the injected holes are moving through the crystal from the emitter point, diffusing as they go. When the holes arrive at the collector, the voltage applied to the oscilloscope rises to a maximum and then falls to its original value when the pulse of holes has passed. This increase in voltage on hole arrival which is made by the circuit to be proportional to the increase in collector current has been shown to be proportional to the hole densities arriving at the collector point (see reference 1). The transit time t for the injected holes is the time represented by the distance measured from the center of the pulse produced by injection to the maximum signal produced by the arrival of the holes.

The potential difference in the germanium crystal between the emitter and collector points may be measured with the potentiometer shown in Fig. 4 and the distance between these points ascertained with a microscope equipped with a micrometer eyepiece.

By definition, the drift mobility is $\mu = v/E$, where v is the average velocity of a charge and E is the electric field. Also $v = l/t$ and $E = V/l$. Substituting these values in the above equation, we obtain an expression for the drift mobility in terms of measurable quantities $\mu = l^2/Vt$.

Since the ratio of the distance to which the holes are translated by the electric field to that to which they diffuse increases as the \sqrt{V} , it is desirable from this point of view to make the voltage between the emitter and collector points as high as possible (see reference 2). On the other hand, high voltage leads to an increased temperature of the crystal. The mobility has been shown to vary as T^{-1} , where T is the absolute temperature. It is therefore desirable to have the sample temperature close to room temperature. Even so, it is necessary to make a rough correction for temperature rise above ambient.

The correction to the increased temperature of the sample is made by making use of the fact that the germanium sample has been coated with a thin layer of Ceresin wax which has a melting point of 51°C. The current through the sample is increased till the wax melts. It is then assumed that the temperature for lower values of current is proportional to the heat dissipated in the crystal or that the increase in the tempera-

ture of the sample above ambient is $\Delta t = k(I)^2$. At the melting point of the wax Δt is known and I is measured so that the proportionality constant k can be evaluated. In the measurements of drift mobility, therefore, the temperature is assumed to be $T = T_0 + KI^2$, where T_0 is the absolute ambient temperature.

The experiment has been described in connection with hole injection into n -type germanium. The apparatus is equally well adapted, however, to the study of injected electrons into p -type germanium. For this purpose, it is only necessary to reverse the polarity of all of the batteries and to apply a negative pulse instead of a positive one to the emitter.

Procedure and Apparatus.—Place an n -type single-crystal rod of germanium in the center of the micromanipulator stage. Bring the contact points of the two bottom micromanipulators in contact with the plated electrodes at the ends of the crystal, using a microscope to insure contact. Using the two upper micromanipulators, place the phosphor bronze points in contact with the germanium crystal surface. The upper right point (collector) should be within 0.1 mm of the sandblasted region at the right end of the rod, and the upper left point (emitter) should be about 7 mm away from the collector (more than 8 mm will be difficult to measure with the microscope provided). Connect the rest of the circuit (excepting the potentiometer) as indicated in Fig. 4. Close the switch of the current-control circuit and, viewing the sample through the microscope, find the current required to just melt the wax. Record the current and the room temperature. Reduce the current to approximately half this value. "Form" the collector point by connecting the clip of the forming circuit (see Fig. 6) to the ground and touching the collector micromanipulator with the other lead while pressing the key. Turn on the oscilloscope and pulse generator. Set the pulse generator at maximum amplitude. Adjust the synchronization on the oscilloscope. If the pattern is noisy and if the signal is insufficient, "form" the collector point again. This may be done while the oscilloscope and pulse generator are operating. When a satisfactory oscillogram is obtained, reduce the amplitude of the pulse generator.

Depress the key on the C. R. O. circuit. This shorts the 1000-mmF condenser. Observe that the noise increases, but that the oscillogram remains unchanged showing that the time constant of the circuit is sufficiently long to prevent signal distortion. It may be observed that the transit time decreases when the current through the crystal is increased. Since this increase in current increases the electric field which is directed from the emitter toward the collector, the injected carriers must be positively charged. Further, since these carriers are shown to have mobilities of the order of several thousand $\text{cm}^2/\text{volt sec}$, they can only be positive holes.

Select a convenient value of current and record the transit time of the holes obtained as the pulse amplitude is decreased to obtain the smallest perceptible signal thus reducing conductivity modulation to a minimum. Record this current. Disconnect the C. R. O. circuit and the pulse generator and connect the potentiometer as shown by the dotted lines of Fig. 4. Measure the voltage V between emitter and collector with the value of current I equal to that used while measuring the transit time. Measure the distance between the emitter and collector points with the microscope.

Repeat the experiment using a p -type germanium sample making certain to reverse all of the battery polarities and apply a negative pulse. Calculate the mobility of electrons and holes in germanium. Reduce the value of mobility obtained to 300°K .

Equipment List.

Pulse generator (see attached circuit)
 Potentiometer
 5000-ohm potentiometer
 500-ohm potentiometer
 10 000-ohm resistance

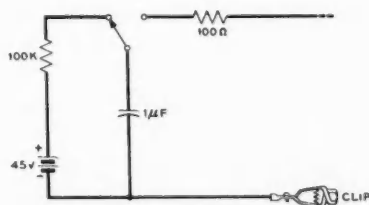


FIG. 6. Forming circuit for use with collector on n -type or emitter on p -type sample.

27-ohm resistance
 Key
 Galvanometer
 22½ volt battery
 Switch
 Current control circuit
 5000-ohm potentiometer
 500-ohm potentiometer
 10 000-ohm resistance
 22½ volt battery
 Switch
 Voltmeter (1, 10 volts)
 Milliammeter (1, 10 ma)
 Cathode Ray Oscilloscope Circuit
 5000-ohm resistance
 1000-mmF condenser (time constant of
 circuit 1×10^{-3} sec)
 Switch
 22½ volt battery
 Oscilloscope
 Tektronix 511AD or equivalent
 4 Micromanipulators mounted on insulating
 base
 Microscope equipped with micrometer eye-
 piece.

Preparation of Germanium Rod for Drift Mobility Experiment (Experiment No. 2) Procedure.—1. Prepare ingot slice for sawing by waxing to saw plate. The rod is to be cut from a 0.045 in. to 0.050 in. thick slice of germanium $\frac{3}{4}$ in. to 1 in. in diameter.

2. Cut rod with diamond saw so that it is 0.045 in. wide. Use care while sawing to cut only 0.003 in. to 0.004 in. for each saw pass.

3. Take the roughed-out rod and wax it to a 0.040 in. lapping block. Grind with No. 800 carborundum until the rod is nearly flush with lapping block and then polish the rod on cloth with polishing alumina. Repeat process on adjacent side.

4. Place rod on 0.035 in. lapping block. Repeat grinding and polishing steps, as in step No. 3, on the remaining sides.

5. Coat polished rods with polystyrene cement except for 4 mm on each end. (Polystyrene cement consists of solid polystyrene dissolved in toluene, the amount of toluene used depending on consistency desired).

6. Sandblast the uncoated ends of the rod.

7. Rhodium plate sandblasted ends of rod for a distance of two mm. The remaining two mm of sandblasted area provides a recombination area to help eliminate carrier injection from the end electrodes. Plate rod at 45° to 51°C in rhodium-plating electrolyte with a current of about 3 ma for about 5 minutes. (See sample preparation data sheet for experiment No. 1 for information on rhodium-plating electrolyte.)

8. Remove polystyrene on middle of rod thoroughly with toluene.

9. Cover electrodes and recombination areas with polystyrene cement. Place rod in platinum-wire holder and fasten with additional polystyrene cement.

10. Etch rod in C. P. 4 etch for 2½ to three minutes. (See data sheet on sample preparation experiment No. 1 for composition.)

11. Rinse rod in distilled water and dry on clean filter paper.

12. Hang platinum holder and rod in toluene to loosen rod. Remove the rod from the holder and rinse thoroughly in toluene again. **MAKE SURE ROD IS VERY CLEAN.**

13. Prepare antimony oxychloride (SbOCl) suspension by placing a few crystals of antimony chloride (SbCl_3) in distilled water.

14. Place a negative electrode of tantalum or platinum in the suspension and connect it to the negative terminal of a 1½ volt cell. Connect the positive terminal to tweezers holding the etched rod. Lower the rod into the suspension until all of the etched surface has been covered. (Do not allow tweezers to touch liquid.) Allow sample to remain in the antimony oxychloride suspension for 5 minutes.

15. Remove rod from suspension, rinse thoroughly, and dry on clean filter paper.

16. Place rod in hot synthetic ceresin wax and allow the sample to come to the temperature of the molten wax.

17. Remove the rod from the wax; and, while the rod is kept warm over the hot plate, remove the excess wax coating by means of a clean filter paper.

The rod is now finished and may be used for mobility measurements.

This concludes experiment No. 2 which has been a direct quotation from the Bell Telephone Laboratories' report.

EXPERIMENT 3

Hall and Suhl Effects in Germanium

The Hall effect, which is well known to most physicists, is one of the fundamental experiments in physics which has advanced our understanding of electrical conductivity possibly more than any other single experiment. In the Hall effect a potential difference is developed across a conductor carrying a current when the conductor is placed in a magnetic field. This potential difference arises because the moving charged particles are deflected in the magnetic field to the sides of the conductor. The charged particles then produce an electric field which is transverse to the sample. In a short interval of time a sufficient number of carriers are deflected to the sides of the sample such that the electric field thus produced exerts a force on the charged particles equal to the transverse force on the particles arising from their motion in the magnetic field. At this time, the current again flows in the same direction it would in the case where no magnetic field is present.

The Hall effect is small for metals, but in the semiconductor, germanium, it is quite large. Attempts have been made to measure the Hall effect resulting from ionic conductivity in ionic crystals; in this case, it should be exceedingly small.

At one time there was considerable concern over the fact that in some cases the Hall effect was positive—that is, the potential difference produced was of opposite sign to what would be expected for electronic carriers. This has been explained by the fact that in these cases the conductivity is caused by holes which have an effective positive charge.

In the first part of the experiment the Hall potential difference in germanium is measured. The same germanium rod used in the mobility experiment is satisfactory. By placing probes on the rod, the transverse potential difference can be measured. The direction of the resultant electric field in the sample can be obtained by adding the transverse electric field and the longitudinal electric field (because of the battery). From the angle θ , between the resultant electric field and the longitudinal electric field, the Hall mobility μ_H can be calculated using the equation

$\theta = \mu_H H / c$, where H is the magnetic field strength, and c is the velocity of light. The details of the calculations are given in the Bell Telephone Laboratories' experiment sheets. The Hall mobility is related to the drift mobility μ_D obtained in experiment 2 by the equation

$$\mu_H = 3\pi\mu_D/8.$$

Aside from its fundamental importance, this experiment gives a student the opportunity to make measurements of such quantities as resistivity and magnetic field strength—measurements which he then uses to obtain other valuable results.

The second part of the experiment deals with the Suhl effect. This effect, like the Hall effect, concerns the motion of charged carriers in magnetic and electric fields. However, there are important differences. The Hall effect concerns the motion of majority carriers, whereas the Suhl effect pertains to the motion of minority carriers in the presence of majority carriers. The minority carriers in *n*-type germanium are holes and in *p*-type germanium they are electrons. In the Hall effect the forces upon the carriers, arising from the transverse electric and magnetic fields, are in opposite directions, whereas in the Suhl effect they are in the same direction. This results in a deflection of the minority carriers to the sides of the sample even in the steady state. When the carriers travel toward the surface they are continually recombining, but most of the recombination will occur at the surface for germanium of reasonably long lifetime. Figure 7(a), by Shockley, shows the motion of the carriers in the case of no magnetic field, and Figs. 7(b) and 7(c) show the motion in a magnetic field.

To observe the Suhl effect, minority carriers are introduced into a germanium rod which is carrying a current of majority carriers. The rod is also subjected to a transverse magnetic field. The experiment amounts to injecting minority carriers into a specimen in which the Hall effect is taking place. The minority carriers are deflected to the surface by the electric and magnetic fields. A maximum current occurs at some particular angle to the longitudinal axis of the sample. By moving a probe along the surface of the germanium, a position can be found where the current is a maximum, thus verifying the

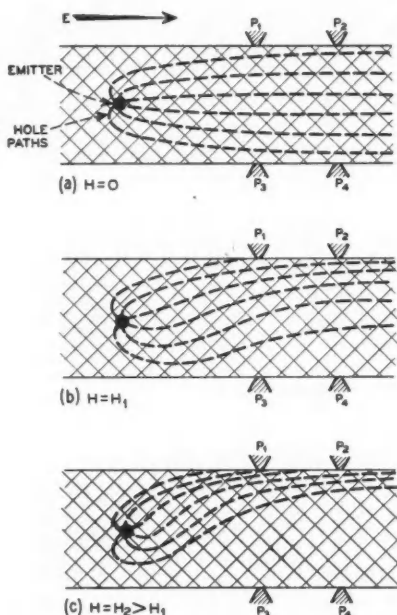


FIG. 7. Paths of holes injected into n -type filament (after Shockley, see reference 1).

theoretical prediction that the magnetic force on the carriers is in the same direction as the electric force.

EXPERIMENT 4

Rectification and Zener Current for p - n Junctions

A p - n junction is the transition region between a p -type and an n -type semiconductor. In this experiment the current-voltage characteristics of a germanium p - n junction are measured for both forward and reverse biases, and the experimental plot is compared with the theoretical. Figure 8 shows the results of such an experiment, demonstrating clearly the rectifying character of a p - n junction. When the reverse bias is increased, it can be seen from the figure that the current rather suddenly becomes very large. This current at its large value is called the "Zener current." It may arise from the electrons tunneling from the valence band into the conduction band. Recent evidence by K. G. McKay,² however, indicates that a cascade process may be responsible in many cases. According to this

² K. G. McKay, Phys. Rev. **94**, 877 (1954).

theory, the holes as well as the electrons are accelerated in the strong electric field of the junction until they gain sufficient energy to produce more electron-hole pairs by collision, thus cascading the process. It is particularly interesting that one can obtain the Zener current with such small differences in potential since one normally associates the Zener current with dielectric breakdown at thousands of volts. However, in the p - n junction one can obtain high-field strengths without having to resort to high differences in potential because almost all of the fall in potential occurs in the transition region which is very thin (around 10^{-4} mm). This is of particular advantage where student safety is of concern. The experiment gives unusual satisfaction to the teacher and student since the theory of the p - n junction is on a sound theoretical basis.

EXPERIMENT 5

Photo Effects in a p - n Junction

When light is incident on p -type germanium, electrons in the valence band are excited into the conduction band, leaving holes in the valence band. The electrons then exist as minority carriers in the conduction band, diffusing through the crystal. In a short time, however, the electrons recombine with the holes. The length of time that the electron exists in the conduction band is related to a time called the lifetime of

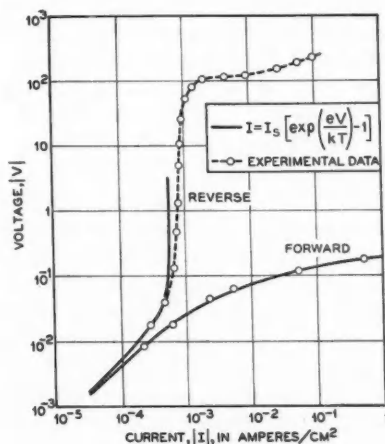


FIG. 8. Theoretical rectification curve and experimental data for a p - n junction.

the electron. Similarly, if n -type germanium is irradiated with light, electrons are excited into the conduction band leaving holes behind. In this case the holes are the minority carriers, and they will recombine with the electrons. A main purpose of this experiment is to measure the lifetime of electrons in p -type germanium and the lifetime of holes in n -type germanium. The lifetimes are determined by a rather ingenious method. A p - n junction is biased in reverse. An exceedingly small image of a light source, obtained by using a microscope objective in reverse, is focused on the surface of the p - n junction. If the light spot is on the p -side of the junction, electrons are excited into the conduction band. With a reverse bias on the junction, the electric field is almost zero everywhere excepting in the narrow transition region shown in Fig. 9. Any electrons that diffuse to the transition region are caught in the strong electric field which forces them to the n -side, thus registering as a current in the external circuit. The farther the light spot is from the junction, the greater is the probability that the electrons will recombine with the holes before they have time to diffuse to the junction; therefore the current increases as the light spot approaches the junction. If the logarithm of the current is plotted as a function of the distance of the light spot from the junction, a straight line is obtained. From the slope of the

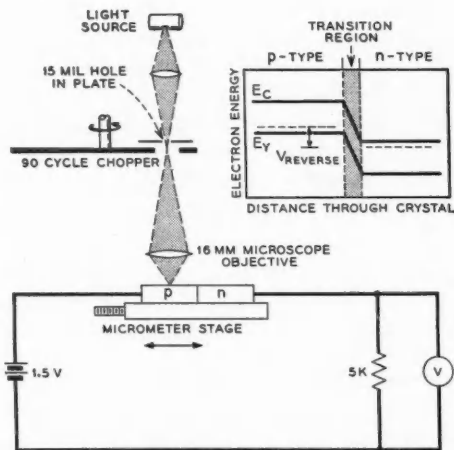


FIG. 9. Circuit and apparatus for determination of lifetimes of minority carriers. Also shown are the energy bands across the junction.

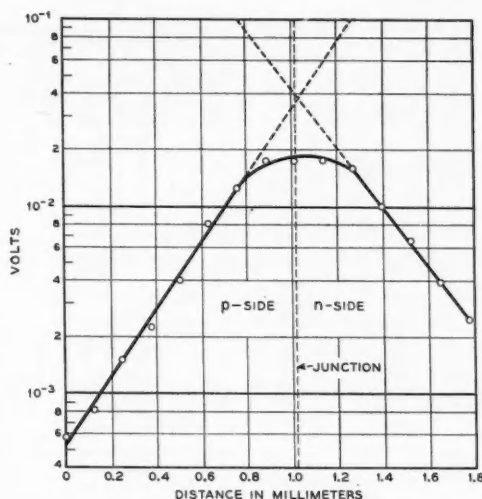


FIG. 10. Difference in potential across the resistor shown in Fig. 9 as a function of the position of the light spot (from a student report at Carnegie Institute of Technology).

curve and the known diffusion constant (which can be obtained from the mobility experiment) the lifetime of the electrons can be determined. The lifetime of the holes can be obtained in a similar fashion. To eliminate the effect of any constant light and the effect of the dark current, an intermittent light source is used, the photocurrent being determined by measuring the ac voltage across the load resistor. Figure 10 shows a typical curve taken directly from a student report at Carnegie Institute of Technology. The lifetime of the holes obtained from this curve is $13.8 \mu\text{sec}$, and the lifetime of the electrons is $8.19 \mu\text{sec}$.

EXPERIMENT 6

Characteristics of an n - p - n Junction Transistor

The junction transistor was proposed by Shockley as a result of theoretical considerations. There are many variations of the junction transistor, but the one investigated here is the n - p - n junction transistor. It consists of two sections of n -type germanium separated by a thin layer of p -type germanium. All three parts, however, are part of the same single crystal. Wire leads are soldered to each of three parts. These contacts are low-resistance ohmic contacts, that is, they offer low resistance to the flow of majority

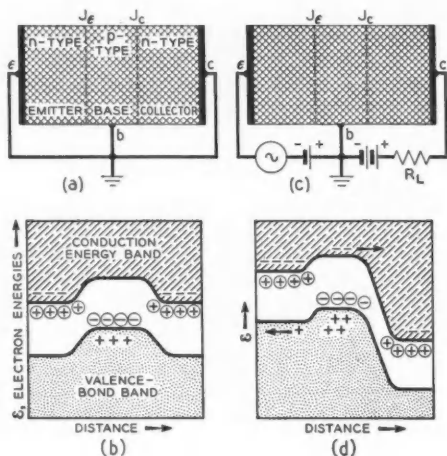


FIG. 11. The n - p - n structure and the energy-level scheme: (a) and (b) thermal equilibrium; (c) and (d) biased as an amplifier. (After Shockley, Bell System Tech. J.)

carriers. The energy bands in the transistor, when no bias is applied, are shown in Fig. 11b. Upon applying a forward bias to the emitter and a reverse bias to the collector, the energy bands are altered (Fig. 11d). The forward bias on the emitter lowers the potential barrier, allowing more electrons to get across the junction. With the collector junction biased in reverse the height of the potential barrier is increased for electrons diffusing from the right to the left, however, the flow of electrons from left to right is aided by the potential drop. The p -type region is made sufficiently thin so that most of the electrons that diffuse into it from the left will diffuse on through into the n -region on the right instead of recombining with the holes. If the conductivity of the n -region is much higher than the conduc-

tivity of the p -region, the current will mainly be a flow of electrons. Since the second junction, which is in the collector circuit, has a high impedance, power and voltage gain are possible. When an ac signal is applied to the emitter circuit, an amplified ac output is obtained from the collector circuit.

In this experiment the collector current is determined as a function of the collector voltage and the emitter bias. This is most readily observed by placing the characteristic curves on an oscilloscope. The circuit parameters

$$\alpha_c = -\left(\frac{\partial i_c}{\partial i_e}\right)_{V_c} \quad \text{and} \quad \alpha_e = -\left(\frac{\partial i_e}{\partial i_c}\right)_{V_e}$$

are obtained. The power gain and the collector saturation current are also determined.

In the second part of the experiment, a very interesting investigation of floating potentials is made. It is found that if the collector circuit is open circuited, the n -region on the collector side will have almost the same potential as the n -region on the emitter side when the emitter bias is varied. This observation is in accord with the theory.

CONCLUSION

All of the experiments which have been discussed were developed by the Bell Telephone Laboratories. The presentation given here was not intended to explain the basic theory involved, but to briefly show the general scope of the experiments. Their true value can be fully appreciated only upon performing the experiments. We wish to thank the Bell Telephone Laboratories for the great help they have given teachers of transistor physics.

A new discovery is important not only in itself, but for the whole new train of work and thought which is opened out by it.

It must be the strength of the traveler and not of the guide that must conquer the difficulties of the journey.—Thomas Young, *Natural Philosophy* by ALEX WOOD (Cambridge University Press, 1954).

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics

65. The Discovery of the Zeeman Effect

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(Received February 19, 1954)

This article reproduces and interprets three stained glass windows that have been installed in the Kamerlingh-Onnes Laboratory at the University of Leiden to commemorate the discovery of the Zeeman effect.

MEMORIALS in stained glass commemorating scientific discoveries are few indeed. It therefore seems appropriate to reproduce in this series the interesting windows in the Kamerlingh-Onnes Laboratory of the University of Leiden which commemorate the discovery of the Zeeman effect. I am greatly indebted to Dr. J. van den Handel for providing me not only with excellent photographs but also with most of the detailed explanation which follows.

The magnetic resolution of the spectrum lines known as the Zeeman effect was discovered in the fall of 1896 and announced to the Amsterdam Academy on October 31st.¹ A theoretical explanation was immediately given by Lorentz² and further experimental verification of the theory were announced on November 28th.³ Pieter Zeeman (1865–1943) was later Professor of Physics at the University of Amsterdam, but at the time of this discovery he was an assistant to Professor Kamerlingh-Onnes at Leiden and was working on the Kerr effect for his doctor's thesis. As he himself has said,⁴ he was stimulated by the reading of Maxwell's sketch of Faraday's life to repeat Faraday's vain attempt of 1862 to detect any change in the lines of the spectrum of a flame when under strong magnetic influence. The Rowland concave grating, invented in 1881, and more powerful magnetic fields were available to him and as a result he succeeded where as remarkable an experimenter as Faraday had failed. Sir Oliver Lodge's philosophical and rather

humorous account of the discovery is probably worth repeating in spite of its length for the benefit of each new generation of physicists. In a lecture entitled, "States of Mind Which Make and Miss Discoveries," delivered to the Institute of Metals,⁵ he spoke as follows:

"The Zeeman effect, too, is rather instructive from my present point of view in its incipient stages. Larmor's theory of radiation, before the era of electrons, had shown virtually that if a source of radiation were plunged in a magnetic field, the lines of the spectrum ought to be broadened, because a radiating atom would be influenced by any magnetic field in which that revolving or vibrating atom constituted an electric current. It was well known that an electric current was perturbed by magnetism, and this perturbation ought to show itself in the lines of the emitted spectrum. Instead, however, of getting an experimenter to try this with modern devices, that is, with a Rowland grating and a very strong field, Larmor—perhaps deterred by the knowledge that Faraday without theoretical clue had looked for some such effect by the aid of prisms and other inadequate devices known in his day, and had failed to find it in spite of his experimental skill—Larmor, I say, proceeded in a state of mind which I may call that of super-theoretical knowledge, to calculate quantitatively how much effect was to be expected; i.e., how much could be expected from any reasonable field, acting, let us say, upon sodium light. He found it surpassingly small, and therefore gave up the quest. He had no idea at that time of anything smaller than an atom that was likely to radiate; and if it were

¹ Verslag. Akad. Wetenschap Amsterdam 5, 181 (1896).

² H. A. Lorentz, *Phil. Mag.* 43, 232 (1897).

³ Verslag. Akad. Wetenschap Amsterdam 5, 242 (1896); see also 6, 13, 99, and 260 (1897) and *Phil. Mag.* 43, 226 (1897).

⁴ Pieter Zeeman, *Nature* 128, 366 (1931).

⁵ Sir Oliver Lodge, *J. Int. Metals* 41, 1 (1929).



FIG. 1. Left-hand panel of Zeeman memorial window in the Kamerlingh-Onnes Laboratory at the University of Leiden (Courtesy of Dr. J. van den Handel).

the whole atom that radiated, the effect of a magnetic field would be hopelessly small; for theory showed that it would depend on the ratio of charge to mass, and the mass of an atom is much too big: nearly 2000 times too big.

"Zeeman, however, undeterred by super-theory and quite independently of it, proceeded to repeat Faraday's old experiment by examining the spectrum of a sodium flame immersed in a strong magnetic field, by means of a Rowland grating. He found the effect—small indeed, but not null; the lines were slightly broadened . . .

"This illustrates that an experimenter should seldom be deterred by a theoretical difficulty; for the data on which the theory is dependent may be erroneous. The theory mathematically may be right enough, but the data, the essential physical machinery, may be different from what had been anticipated. Zeeman's effect was observable because the radiating particle was a unit electric charge, and because the minute mass of the electron was soon afterwards brilliantly ascertained by Sir. J. J. Thomson; and so the outstanding difficulty was removed. The magnitude of Zeeman's effect, in fact, proved that the electron was the real radiator—a far-reaching discovery applicable to light of all kinds.

"Then came H. A. Lorentz, who applied his super-theoretical knowledge to an electron revolving like an electric current in a magnetic field; and predicted that the lines should not merely be broadened but should be doubled or trebled according to the way the source of light

was looked at, whether along the lines of force or across them, and further, that the lines would be polarized in certain definite fashion. Zeeman immediately proceeded to re-examine the phenomenon with still greater power, and verified every detail of Lorentz's prediction, subsequently finding out many other details about the lines—one of the sodium *D*-lines being quadrupled, and the other sextupled, for instance; all which is now explained by a further elaborated theory, and has become a part of modern physics."

This story is told symbolically in the three stained glass windows which are here reproduced. The left window (Fig. 1) shows Zeeman making the original discovery as reported on October 31st (31 X). A sodium flame stands between the polepieces of an electromagnet (lower right). The light beam traverses a slit (lower center) and is dispersed by means of a Rowland grating (lower left). The broadening observed is indicated in the upper center, while at the upper right a switch is shown.

The central panel (Fig. 2) symbolizes the theoretical part of the work. Lorentz (center) postulated that an atom contains elastically bound moving charged particles (left) and derived the expression $\pm \frac{1}{2} h e / m$ for the broadening. From this concept he also derived the existence of a longitudinal and a transverse effect giving rise to a doublet and a triplet (extreme right). In the longitudinal case he predicted the difference in circular polarization at the borders of the broadened line (the apparatus was not yet



FIG. 2. Center panel of Zeeman memorial window (Courtesy of Dr. J. van den Handel).

powerful enough to show a splitting). Also he pointed out that it would be possible to calculate the ratio of the masses of the electron and the proton (to use the modern names).

The right-hand panel (Fig. 3) shows the confirmation of these predictions on the 28th of November (28 XI). A quarter-wave plate and a Nicol prism is inserted between the grating and the eyepiece. The ring with the divisions can be used for the adjustment of the quarter-wave plate. Zeeman can control the reversing switch (lower right) and the quarter-wave plate and Nicol prism while looking through the eyepiece. In one position of the Nicol and switch, one border of the broadened line is extinguished; when the Nicol is turned through 90° or when the current through the magnet is reversed, the other border disappears. This is represented in the two semicircular diagrams at the lower left.

The Latin dedication above the panels may be freely translated as follows: "These panels have been dedicated to Peter Zeeman; they commemorate the fortunate discovery which became known as the Zeeman effect and which, together with the ideas on light conceived by Lorentz, contributed to experimental and theoretical research and offered a glorious example

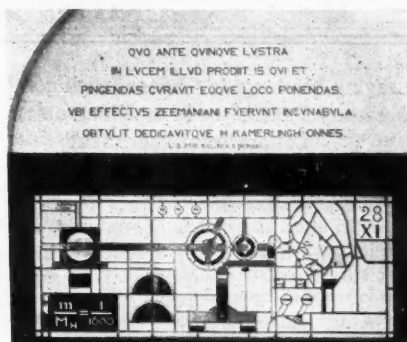


FIG. 3. Right-hand panel of Zeeman memorial window (Courtesy of Dr. J. van den Handel).

of the power of cooperative efforts in lifting the spirit; presented and dedicated by H. Kamerlingh-Onnes on the same day, the eve of the kalends of November, on which twenty-five years earlier, these discoveries were published; H. Kamerlingh-Onnes caused these panels to be painted and installed in the very place where the cradle of the Zeeman effect stood. Leiden, October 31, 1921."

As indicated in the upper left hand corner of the third panel, the artist was Harm K. Onnes, Professor Kamerlingh-Onnes' nephew.

New Members of the Association

The following persons have been made members or junior members of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Phys.* **22**, 506 (1954)].

Active Members

Avery, Madalyn, Kansas State College, Manhattan, Kansas
 Baranelli, Michael, 106 Cotton Street, Birmingham 11, Ala.
 Burkart, Robert P., 2175 35th Street, Los Alamos, N. M.
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(Continued on page 654)

NOTES AND DISCUSSION

Encouraging Students to Choose Intelligently an Arbitrary Parameter in an Elementary Laboratory Experiment

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STUDENTS often blame their apparatus, rather than their judgment, for poor results. The simple magnetometer affords a good introduction to the analysis of apparatus in terms of expected uncertainties, because there is involved the arbitrary selection of an important parameter.

The standard experiment in the elementary laboratory consists of the determination of the horizontal component H of the earth's magnetic field. The value of H is calculated by the solution of two simultaneous equations in the unknowns H and M (the magnetic moment of the bar magnet). The first of these equations, involving the use of the bar magnet as a torsion pendulum, requires no comment. But, in the experimental setup for obtaining data for the other equation, the student has a real opportunity to exercise judgment. The instructions are usually about as follows: measure the angle θ (Fig. 1) four times for a given distance x ; this is done by making two readings of θ with the bar magnet East of the compass needle (by reversing the magnet end for end), and two readings in the same way with the magnet West of the compass. The value of θ to be used in the calculation is the average of the four.

The parameter left to the discretion of the student is the value of x . Some students will select a small value of x , producing a value of θ nearly 90° . Others will place the magnet far enough away to make the angle θ nearly zero. Most will not worry about it at all, but choose "any" value of x , provided only that something comes out "even."

If the instructor is willing to devote a few minutes to the encouragement of experimental skill and simultaneously reduce by a small amount the degree of "cook book" procedure, he can present a quite simple (indeed, over-simplified) error analysis. An inspection of the apparatus will usually bring unanimous agreement that θ is the most uncertain quantity to be measured, since the compass needle is small and the protractor correspondingly small. Student estimates of the uncertainty are of the order of 1° to 5° . For the moment, avoid the question of uncertainty in any other data.

The equation for $\tan\theta$ for the Gauss A position is supposed to have been learned. Explain that the approximation $\tan\theta = 2M/Hx^3$ is valid in the error analysis, even if it is not always sufficiently accurate for the actual calculation of H . Therefore, the problem reduces to the evaluation of $x^3 \tan\theta$, and the question arises as to the optimum value of θ . The following four analyses are given without comment, and each student is asked to select what he considers the best choice.

(a). Since it is the fractional error in θ that is important, θ should be as large as possible, and therefore x should be as small as possible. For example, if θ is near 90° , a 2° error is only about 2 percent.

(b). The rate of change of the tangent is smallest at $\theta=0$. Therefore, θ should be as small as possible, and x as large as possible. Quantitatively, $d(\tan\theta)/d\theta = \sec^2\theta$, and this is a minimum at $\theta=0$.

(c). It is not θ , but the rate of change of θ with x , that is important, and this rate of change should be a maximum for best accuracy. Thus, we have $x^3 \tan\theta = \text{constant}$, from which $3x^2 \tan\theta dx + x^3 \sec^2\theta d\theta = 0$. Solve for $d\theta/dx$, then find $d^2\theta/dx^2$ and set it equal to zero. The result becomes $\tan\theta = \sqrt{2}$, from which $\theta \cong 55^\circ$. Therefore, we should adjust x so that θ is about 55° .

(d). Since it is $\tan\theta$ that enters the equation, we should make the fractional change in $\tan\theta$ a minimum. That is, the error is $d(\tan\theta)/\tan\theta = (2d\theta)/\sin 2\theta$. This quantity is obviously a minimum when $\theta = 45^\circ$.

The ensuing discussion should prove lively, since the reasoning involves what students like to call "common sense," which most first-year students think they have much of. If the majority of the class is finally convinced

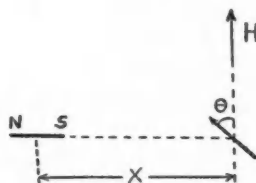


FIG. 1. A bar magnet NS causes a deflection θ of a magnetometer needle which would otherwise lie in the direction of the earth's field H .

that choice (d) is superior, it becomes simple to predict the error to be expected from a given error in θ . Thus, when $\theta = 45^\circ$, the fractional error in $\tan\theta$ becomes $2d\theta$, or about 3.5 percent for each degree error in θ . If 2° is accepted as the expected error, the resulting error in M/H is some 7 percent.

The analysis has assumed that the uncertainty in x is relatively small. This will be true in general if the moment of the bar magnet is not too small. In the usual experiment, x will be about 20 cm for $\theta = 45^\circ$. A 1-mm error in determination of x will cause about 1.5 percent error in x^3 . Thus, the assumption that θ is likely to be the critical measurement was justified.

For students with slightly more mathematical maturity, the matter can be summed up in more general form. Let $x^3 \tan\theta = f(x, \theta)$. Then, we require that df/f shall be a minimum. Therefore, we have

$$df/f = \frac{\partial f}{\partial x} dx/f + \frac{\partial f}{\partial \theta} d\theta/f = \frac{3}{x} dx + \frac{2}{\sin 2\theta} d\theta.$$

The details of the treatment follow directly from this equation.

LETTERS TO THE EDITOR

A Philosophy of Demonstration Experiments

IN this paper I discuss an attitude regarding demonstration experiments—a point of view—or, quite properly, a philosophy. My remarks grow out of observation and out of discussion with teachers and students. The problem is best delineated by relating the following episode. On a certain day I encountered a professor examining a demonstration arrangement which had been set up for him. He pondered over the equipment and finally commented, "I don't believe I shall use it after all. It didn't work too well the last time." At this place I hesitatingly suggested that it was still a *good* demonstration, even if it did not work. Indeed, I dared to suggest that it served a better purpose by not working. But the professor was adamant in his own position. "No, indeed," he countered, "the experiment must demonstrate a principle and if it does not work it is useless." This scene gives us the point of departure for this note.

We are agreed that a demonstration is intended to show a certain phenomenon. We discuss the situation on the blackboard; we show a diagram; we establish what happens mathematically. If, now, we reproduce this pictorial and linguistic arrangement *physically* we are in a position to predict how nature will behave. But the requirements of nature are painfully demanding. Unless we fulfill her every requirement the experiment either fails or is at best a gross approximation. If the experiment succeeds and if our prediction is fulfilled, we acquire quickly a smug sense of satisfaction, but, quite frankly, we have no more than we expected. But this success is, in my opinion, like that borne out by the classical phrase. The operation was a success but the patient died! By this I mean that invariably the student ceases to think about the experiment. He knew what was expected and it happened. There is not much really exciting about that, unless, of course, you have a measure of drama in your soul and every experiment stirs you inside. Unhappily, few students have this "feel" for the phenomena of nature and far too few teachers play up the drama which so abounds in nature. An experiment which always works is indeed a good experiment, but I make a case here for that demonstration which does not always work. In truth, I prefer often to allow the demonstration not to work, whereupon we are required to explain its failure. Invariably much more physics is encountered if it does not work. At least the student is incited to some thinking. In addition, certain physical phenomena are often revealed when a demonstration fails to work as predicted, phenomena which would be altogether obscured had the demonstration worked.

There is now another point of view which I feel stirred to make regarding demonstration experiments. This has been spoken of before. It is this: *a demonstration is for the student*. Too often—much much too often—the professor shows the demonstration and talks the demonstration as

if the student knew a goodly bit about it already. This is not so. I have examined the situation I here report on and the evidence abounds that students generally leave a demonstration not too certain of what happened. The professor either takes too much for granted and leaves too much unsaid or he talks mostly to himself—in a monotone that wants for life—or the students cannot see what is going on. It is regrettable that so much effort and time and labor are wasted on the professor!

As evidence of what I mean let me report on my findings by way of one case. I have made it a habit, over the years, to have on my desk a tall glass cylinder partly filled with water and in which resides the classical Cartesian diver. A rubber membrane is drawn across the top and the gadget irresistibly and unerringly invites a visiting student to "play with it." Invariably I put some leading question which brings from the student some reply as "It's really interesting, isn't it, how that thing goes down." What astonishes me is that not too many students tell me *precisely* what sends the diver down. They have all seen the device. Hardly a professor lives who has not shown it. But few, very few, know the mechanism by which it sinks.

The moral of all this is clear. A demonstration must be prefaced by a clear statement of what we have in hand, what we propose to do with it, what forces act and what results we can expect. And all must see it—not just those in the front row! If now, a bit of guile and deception can be introduced so that predicted results do not at first appear, all the better. Indeed, if the experiment possesses intrigue of its own, that is, if its performance cannot be predicted with irrevocable law, the professor should still show it. It is more interesting this way.

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Millikan Oil-Drop Experiment— A Demonstration Model.

A DEMONSTRATION model of the famous Millikan Oil-Drop experiment was exhibited at the Iowa Colloquium of College Physicists. The role of the oil drop was played by a helium-filled balloon. The condenser was a box-like affair with plexiglass walls on three sides $\frac{1}{2}$ meter high and with top and bottom plates of 24-gauge sheet metal 2 ft square. The potential difference between the plates was variable from 1000 to 10 000 volts, obtained from an rf oscillator and rectifier.

The operation of the demonstration is delicate in that both the weight of the balloon and the charge on the balloon must be carefully adjusted. In free fall the balloon may either rise or fall slowly, about 5 cm/sec. A thread may be laid on top of the balloon or part of the neck snipped off

as a last minute weight adjustment. The charge on the balloon must be rather small or the image force will hold it to either plate. The charge is about right if the balloon is attracted slowly toward the hand held 1 or 2 inches from the balloon. Moreover, space for maneuvering the balloon is desirable because the equilibrium is unstable due to the image forces.

A measurement of the free fall velocity, the radius of the balloon, and the equilibrium potential yields the charge on the balloon, which may turn out to be several million electrons. Obviously the addition of 1 or 2 electrons can not be detected, but this difficulty only serves to emphasize the genius of Millikan's experiment.

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ARTHUR G. ROUSE

Terms of Statistical Mechanics

THE term *statistical mechanics* was used by Gibbs to designate the determination of thermal properties of systems by means of ensembles of systems. Since the publication in 1902 of his now-classic work, the terms *canonical ensemble* and *microcanonical ensemble* have been generally used by workers in the field. The quantum-mechanical analog of the classical canonical ensemble has also been designated by Gibbs' terms, as is illustrated by the writings of J. von Neumann and Dirac. It is therefore surprising to find new terminology introduced in a recently published *Elements of Statistical Mechanics*, by D. ter Haar. This author uses the term *macrocanonical* as replacement for Gibbs' generally accepted term *canonical*.

It may be argued that this departure from long-established and current practice disregards the historic rights of an individual to name his own discovery, and will confuse students who wish to consult other works in the field. Additionally, the new terminology is not suited to the subject.

It may be presumed that Gibbs used the term *canonical ensemble* in order to indicate a preferential status. A canonical ensemble consists of a continuous distribution of systems that is defined by an exponential function of the energy. The practical value of the canonical ensemble resides in the circumstance that for systems of many degrees of freedom it substitutes for an ensemble in which all systems have the same energy. The latter ensemble Gibbs called *microcanonical*, a term compounded out of *micro* and *canonical*, and which expresses the fact that a canonical ensemble can be viewed as constituted of microcanonical ones.

Dr. ter Haar has now created a new class of canonical ensembles in which he distinguishes between macrocanonical and microcanonical ensembles. It may be conjectured that this new terminology has been patterned after the distinction between macroscopic and microscopic, which was made current by Lorentz and Planck. But the systems included in canonical and microcanonical ensembles by Gibbs are all macroscopic systems and hence the usual

distinction between *macro* and *micro* is not applicable thereto. In fact, one mode of interpreting the canonical ensemble is to view it as an ensemble of subsystems of a microcanonical one. Also, one would expect from the new terminology that since canonical ensembles occur in two types, as designated by the prefixes *macro* and *micro*, the microcanonical ensemble would be the Maxwell-Boltzmann distribution of molecules within an individual system of an ensemble.

In addition to the inappropriateness of the new terminology, its application is not consistent. Dr. ter Haar gives much space to ensembles in which systems have a variable number of particles. Such ensembles were called *grand ensembles* by Gibbs, in order to distinguish them from *petit ensembles* in which systems have a fixed number of particles. Dr. ter Haar retains the term canonical grand ensemble of Gibbs. But if the canonical petit ensemble is to be called macrocanonical, it is difficult to see why the term macrocanonical is not applied to the *canonical grand ensemble* as well. Indeed, the term macrocanonical might seem more appropriate for the canonical grand ensemble, since it in turn can be conceived as an ensemble of canonical petit ensembles.

It would appear that Gibbs' generally accepted terms and distinction between canonical and microcanonical ensembles should continue in use.

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V. F. LENZEN

Terminology of Statistical Mechanics

DR. LENZEN¹ objects to the introduction of the term *macrocanonical* for ensembles called by Gibbs and the majority of other authors *canonical*. I would like to use the opportunity afforded me by the Editor of the *American Journal of Physics* to answer Dr. Lenzen's arguments and at the same time to give the reasons for the introduction of this new term, which I have used before.²

The introduction of a new term does not seem to me to be a major crime and it is interesting to note that Gibbs himself was guilty of this act when he introduced the now standard term *microcanonical* for the energy shell or ergodic ensembles which had been introduced by Boltzmann and Maxwell (for an account see the review article of the Ehrenfests).³

In using the term *macrocanonical* I followed my teacher, H. A. Kramers (and possibly Ehrenfest). The distinction between macrocanonical and microcanonical ensembles is not patterned after that between macroscopic and microscopic, but after Gibbs' distinction between grand and petit ensembles. Just as the transition from grand to petit ensembles is due to a restriction of the number of particles to a fixed value, so the transition from macrocanonical to microcanonical ensembles is due to the restriction of the energy to a fixed value.

A second point is that the use of canonical instead of macrocanonical is not quite logical if one considers the connection with microcanonical ensembles. The prefix

micro- usually indicates small, but a microcanonical is not a small (macro) canonical ensemble.

I do not think that the term *macrocanonical* is likely to confuse students. I have been careful to point out that macrocanonical ensembles are usually called canonical ensembles.

As to the canonical grand ensembles, a moment's reflection will show that for physical reasons grand ensembles will never be microcanonical. The adjective canonical describes this kind of ensemble adequately, even though the use of the term macrocanonical might have been more consistent.

In conclusion I should like to stress that I do not consider the introduction of the term macrocanonical of great importance and if the majority of physicists in the field agree with Dr. Lenzen I shall be glad to toe the line. I would like to thank Dr. Lenzen for giving me the opportunity to give the reasons for introducing this term.

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St. Andrews University
Scotland

¹ V. F. Lenzen, *Am. J. Phys.* **22**, 638 (1954) (preceding letter).

² D. ter Haar, *Proc. Roy. Soc. (London)* **A212**, 552 (1952).

³ P. Ehrenfest and T. Ehrenfest, *Enzykl. Math. Wiss.* **4**, part 32 (1911).

ANNOUNCEMENTS AND NEWS

Training of College Physics Laboratory Assistants

APPROXIMATELY forty men representing thirty-four universities and colleges in the United States met at Northwestern University for a two-day conference, June 25 and 26, 1954, to discuss the training of graduate student laboratory assistants. Nearly all the men present were physics department staff members who are responsible for the work of the college physics laboratory at their respective universities.

The first session was devoted to the topic, *Our Present Training Procedure*. Papers were given by representatives from Pennsylvania State University, Case Institute of Technology, University of Southern California, University of Minnesota, Massachusetts Institute of Technology, and Northwestern University. These papers were followed by reports prepared by the laboratory assistants, at the last three colleges mentioned, on, *How Does This Training Meet the Needs of the Laboratory Assistants?* The purpose of this program was to give the participants a picture of the present procedure in laboratory training and the reaction or viewpoint of the assistants receiving this training.

At the first luncheon meeting, Professor B. J. Underwood, a member of the Northwestern University Psychology Department spoke on *The Student, the Experiment, and the Laboratory Assistant*. Two papers were presented, in the afternoon session, each followed by group discussion. The first of these was, *What Purpose Does the College Laboratory Serve?* by Professor V. E. Eaton of Wesleyan University, essentially a report on a session of the Iowa Colloquium held a week earlier. The second paper, by Professor Eric Rogers, of Princeton University, was titled, *It's Your Experiment*. He discussed a laboratory procedure in which the student plans his own attack on a specific laboratory problem.

The dinner meeting was planned to give the participants in the conference the viewpoint of the college administrator with reference to laboratory problems. Dean T. H. Osgood

of the Graduate School of Michigan State College spoke on, *An Administrator Examines Laboratory Instruction*. Professor O. W. Eshbach, Dean of the Northwestern Technological Institute from 1939 to 1953, discussed the subject, *The Purpose of the College Physics Laboratory in Engineering Education*.

The second morning session was a panel discussion by representatives from the Georgia Institute of Technology, and Stanford, Yale and Notre Dame Universities on, *The Content of the Training Program*, namely, what do we consider essential topics to be discussed in the training sessions with the laboratory assistants? This was followed by a group discussion on special problems submitted in advance by the participants. Two major reports were given. Professor W. C. Kelley, of the University of Pittsburgh presented a paper on *Cooperation and Incentives in the Training of Graduate Assistants in Physics*, and, Professor P. Kirkpatrick, of Stanford University discussed, *The Selection of Laboratory Assistants*.

The paper at the second luncheon meeting was given by Professor M. W. White of Pennsylvania State University, President of the American Association of Physics Teachers. He spoke on, *Organized Laboratory Manuals as Aids for Training Assistants*. The final afternoon session included a panel discussion by representatives from the University of Virginia, Northwestern University Tech Institute, Texas A and M, and the University of California: *Procedure and Recognition of the Training Program and a Report on the Committee on Conclusions*, given by the chairman of the committee, Professor S. C. Brown of Massachusetts Institute of Technology.

A comprehensive report of this conference including major portions of the prepared papers and the group discussions is in the process of preparation. A limited number of copies will be available for free distribution. Requests should be directed to Dr. C. J. Overbeck, Department of Physics, Northwestern University, Evanston, Illinois. These proceedings and a portion of the expenses of the con-

ference were financed by a grant of the National Science Foundation.

In the selection of universities to participate in this conference an attempt was made to contact indirectly the largest number of assistants by inviting the physics staff member responsible for the college laboratory work from those colleges having the largest enrollment in College Physics. A good geographical distribution in the United States was a second consideration.

Some of the conclusions gathered from this group indicate that the most common present practice is for the staff member in charge of the laboratory to have one-hour weekly meetings with his assistants to discuss laboratory problems and procedures. Thus, the novice teachers are given suggestions and help in their new job of teaching students.

Approximately one-third of the colleges represented also hold a series of meetings with the assistants, covering one to five days, in the week prior to the beginning of classes. It was the conclusion of the conference that more time should be used for this purpose in the interest of improving the laboratory work.

Only a few colleges hold post-term meetings with the assistants to analyze the problems of the preceding term. This is another area of our present practice that needs improvement.

The student load per instructor in the laboratory was found to vary from a minimum of twelve to a maximum of about thirty. The most usual load is twenty students per instructor in the laboratory. A higher ratio was considered unsatisfactory educationally. The difficulty of obtaining good staff was expressed as the main reason for the higher ratio.

Four of the most common criticisms of the content and procedure of our present training program and laboratory practices appear to be:

(a) We do not have a clear idea of the prime goal of operating laboratories. What justifies the time, the energy, and the cost of laboratory work? This question is repeatedly asked by both the assistant and the student. A series of recognized goals were presented and evaluated. The two which ranked highest were: "A better and longer lasting understanding of physical principles," and "Experience with and appreciation of the various methods used in experimental science." Certainly we need to know the goals to help effectively direct our efforts in the laboratory.

(b) We permit the laboratory assistant to conduct the laboratory work without insisting that he be thoroughly familiar with the experiment, the apparatus, and the procedure. In a check of present practice it was noted that of the universities and colleges represented at the conference only three *required and checked* to see that all teaching assistants did the laboratory experiment before directing it in the laboratory. Twenty-one additional colleges *requested* that the laboratory assistants do each experiment before conducting it but did not check to see that this request was carried out. A few colleges indicated that they did not request their laboratory assistants to perform the experiment before meeting the laboratory

class. It is interesting to note that after this matter was discussed, twenty of the representatives present indicated they planned in the future to emphasize much more strongly this particular requirement. Several excellent methods of handling this problem were discussed in the conference. One in particular not only insured that the assistants became familiar with the experiment but also provided data to determine how the apparatus worked, its limit of accuracy, etc.

(c) We fail to develop an *esprit de corps* in our group of assistants. It is most necessary that we show the laboratory assistant that his work in the laboratory is an important part of the educational process. He must feel that what *he* is doing is important or his efforts become perfunctory. He should be asked to take part, as a regular member of the teaching group, in the administration and the planning of the laboratory work.

(d) Too little attention is given towards helping the assistant correct his errors and improve his work from term to term. A method of evaluation of this work, now successfully used by several colleges, was discussed. This received enthusiastic comment. The representatives from other colleges indicated that they planned to try this method.

Many general problems which have a bearing on the theme of the conference were also discussed. These included:

- Selection of good teaching assistants.
- Pay increases for demonstrated superior performance.
- Promotion to recitation work.
- Teaching by asking questions.
- Academic credit.
- Uniformity of grading.
- Student-instructor relationship.
- Training by observation.
- Conduct of laboratory work by senior staff members.

The spirit of the conference was excellent, the discussions were informal and vigorous. They were not without humor, yet the theme and business of the conference were maintained. The expressions of the participants indicated that our thinking together and sharing of experiences during those two days provided solutions to many of their problems. We hope that through the *Proceedings* many others, who could not be with us, may share in this experience.

C. J. OVERBECK

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Book Reviews

Physics of Experimental Method. H. J. J. BRADDICK.
Pp. 404+xx, Figs. 148. John Wiley & Sons, Inc.,
New York, 1954. Price \$7.00.

The book of H. J. J. Braddick, well known by his research in the field of cosmic rays, is intended to teach the student some of the principles of experiment, that is, the technique of measurements and their mathematical evaluation. A book of about 400 pages is not expected to cover

the ground in the exhaustive style of a handbook. Instead it wants to give a survey of the available techniques to the senior or first-year graduate student and train him in the understanding of such techniques. This is the meaning of the title, which to some American readers may not convey a clear idea of the book's purpose.

The book begins with a treatment of errors (23 pages). This is followed by an appendix, contributed by J. Maddox, on numerical calculations. Entirely different is the character of the next sections on mechanical design and materials (88 pages). Every beginning research student who worries about instrumentation should study these extremely useful sections before he starts building apparatus. If the same inexperienced student wants to produce high vacuum, he will study the next 34 pages on vacuum technique, but will do well to search more comprehensive books for the indispensable detailed advice.

The reviewer feels that the next sections on electrical measurements (27 pages) and electronics (48 pages) are less needed considering the abundance of books and special courses on these subjects. In a second edition it may be desirable to give more numerical advice.

The chapter on optics is full of useful, practical advice on optical instruments and the technique of photography. Of great interest is the section "Natural Limits of Measurements," which introduces the beginner to the problem of noise. The last 50 pages on "Some Techniques of Nuclear Physics" include a discussion of nuclear track emulsion in addition to the obvious topics.

Very useful are the numerous surveys given by tables, for example, on the combination of errors, loaded beams, solders, insulators, light sources, photographic lenses, types of glass, and other topics.

The mathematical sections of the book are so concise that the student may be inclined to consult additional, more extensive books. Eight pages of references help the student to find such additional advice. The book, which originated from a lecture course given at the University of Manchester, will be very useful to many beginning experimental physicists. A foreword is written by P. M. S. Blackett.

OTTO OLDENBERG
Harvard University

Elements of Statistical Mechanics. D. TER HAAR. Pp. 468+xix. 23×15 cm. Rinehart and Company, New York, 1954. Price \$8.50.

The author of this book is an enthusiastic young theoretical physicist from Leiden, Holland, who spent a year or more at Purdue University, and is now located at St. Andrews University, Scotland. The text of the book was started in Leiden during World War II, continued at Purdue and completed at St. Andrews. Large parts of the book are based on lectures given at Leiden by Professor Kramers, and much of the material goes directly back via Ehrenfest to Boltzmann. The book is intended as a text for graduate students and as a research tool; it is in general too advanced for an introductory course in the U. S. A., but will be an excellent source book for Ph.D. candidates.

The work is divided into four parts. Part A contains a rather rapid review of the more elementary parts of statistical mechanics up to and including the Bose-Einstein and Fermi-Dirac statistics of independent particles. Part B is devoted to ensemble theory, including petit and grand ensembles, and ensembles in quantum statistics. This last topic is discussed directly in terms of the von Neumann density matrix and the Klein-Jordan-Wigner number operators, which are assumed to be already well-known to the student. Part C, Applications, covers a wide variety of fields. The chapter on the Equation of State contains a very useful outline of the classical theory of a nonideal gas, the second and third virial coefficients, and the law of corresponding states; the same topics are discussed in detail for quantum statistics, with a special section on the equation of state for helium gas. Under the heading "Condensation," is a discussion of the Einstein condensation and its relation to the lambda transition in liquid helium, an account of the liquid drop or cluster theory of condensation, and a very brief statement about the Mayer theory. The account of the liquid drop theory is the more welcome because it is based on a particular form of the theory published in a Norwegian journal and not easily accessible to general readers in the U. S. There are two brief chapters on the elementary theory of electrons in metals and on semiconductors. The chapter on cooperative phenomena, besides the familiar discussion of order-disorder problems, includes a discussion of the Lennard-Jones and Devonshire theory of melting, and some exact results for the two-dimensional square lattice. Three short but illuminating chapters on statistical methods in nuclear physics, the origin of the chemical elements, and the theory of rubber elasticity complete this part. There are over one hundred pages of appendices, most of which one feels might easily have found a place in the body of the text. An excellent discussion of the H-theorem and the ergodic hypothesis including both classical and quantum-statistical approaches, occupies fifty pages; there is a brief but useful summary of the theory of irreversible processes, a short discussion of Nernst's third law of thermodynamics and the chemical constants, and a short account of the Darwin-Fowler method of steepest descents. Appendix V discusses the quantum-mechanical theory of intermolecular forces. In Appendix VI are derived the basic statistical formulae valid when relativistic effects are not negligible. A Table of Physical Constants seems to have become a welcome addition to every physics text, and this one is no exception. The mathematical appendix contains little that would not already be familiar to any student sufficiently advanced to be able to read the main text material, but there is of course no harm in having it repeated here. There is a complete glossary of symbols. The latter are well chosen and the typesetting is excellent. A particularly acceptable feature of the book is the comprehensive set of bibliographical notes at the end of every chapter.

In his preface the author asks that his reviewers will let him have the benefit of detailed criticism. The only major criticism that this reviewer has is a complaint over the absence of any set exercises for students; but since the work is not intended as a beginning course, this complaint

need not be given much weight. The author apologizes for the possible foreign flavor of his English. In fact his style is remarkably clear. It is true that the word order in one or two places does not conform quite with English usage; but there is no harm, surely, in this occasional reminder that we owe much to continental European culture for the development of this whole subject.

In this reviewer's judgment, ter Haar's book is a worthy addition to the comparatively small number of good texts on statistical mechanics.

WILLIAM BAND
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Acoustics. T. M. YARWOOD. Pp. 356+x, Figs. 160, 14×20 cm. St. Martins Press, Inc., New York, 1953. Price \$3.50.

The phenomena of acoustics are surveyed with remarkable completeness in this pocket-size book. Except for a few pages principally near the beginning and in the Appendix where the notations of trigonometry and differential calculus appear, the descriptive aspects of acoustics are made fairly easy reading for the student with little mathematical experience. Some previous knowledge of the elementary terms of physics is required. Most necessary formulas, such as the one for the frequency of the vibrating string, are simply stated without proof. Some 200 questions graded as to difficulty appear at the end of the book.

The final chapters on acoustics of buildings and noise and sound insulation (brief though they are) bespeak of a modern outlook on the subjects; the concluding plea for the interest of the young engineer in the problems of aircraft noise is certainly in order. The ultrasonic detection of flaws is described early in the book; immediately following is the intriguing story of how bats use sound to avoid obstacles while flying in the darkness.

The first chapter may prove hard reading for the uninitiated because of the scope of subjects covered: transverse and longitudinal wave motion, intensity of sound (meters are said to be available for measuring intensity!), acoustic filtration, the ear and its sensitivity, microphones, sonic and ultrasonic sounds. These are not simply introductory paragraphs, but contain details which one might expect to find with later discussions of related subjects.

It is worthy to note that underwater sound is described more fully than has been customary in brief elementary texts in the past. Photographs illustrate the expanding bubble about an explosion, transducers designed especially for underwater use are described, and good accounts are given of echo sounding, the design of acoustic mines and sweeping such mines, and the detection of submarines.

Microphones and loud-speakers receive rather parenthetical attention. The concluding section in the chapter on the vibration of membranes, diaphragms, and plates is devoted to microphones. The only index entry on loud-speakers leads to the exponential horn which is concealed in a chapter which treats tuning forks on resonance boxes, open and closed pipes, end corrections, organ pipes, an electronic organ, and the theory of the resonator.

Frequency is noted as a fundamental property of wave motion, meriting a chapter of its own. Considerable space is devoted to the tricks of the nineteenth century, such as the monochord, siren, and falling plate. The quartz crystal, the electronic frequency meter, the frequency bridge, and counter circuits of the modern laboratory are overlooked completely and buried on the final page of the appendix is the brief description of the use of the cathode-ray oscilloscope for frequency comparison.

The older history of acoustics is outlined reasonably well in regard to national origins. Current illustrations, however, seem unnecessarily restricted to the author's own Britain. For example, only one type of electric organ is described, whereas descriptions of types which originated in other countries could well have been used to round out the story. The German part in the development of magnetic recording is ignored. Piezoelectric transducers for underwater use are described rather narrowly—"The earliest kind of oscillator of this type was made of mica, but nowadays quartz is generally used"—in spite of the frequent use of Rochelle salt for this purpose. The Langevin contribution to echo sounding is not mentioned.

Chapter VI on the characteristics of musical sounds starts out by noting that "frequency is an *objective* rate of vibration whereas pitch is a *subjective* sensation by which a listener classifies a note as high or low, and although the two terms are often used interchangeably it is possible for two *pure* notes of slightly different frequencies to have practically the same pitch. This is due to what is called the *differential frequency sensitivity* of the ear. . . ." It is curious that the Shower and Biddulph experiment is cited in support of the distinction between pitch and frequency, whereas the change of pitch which may accompany a change in loudness (frequency being held constant) is not mentioned at all. The items discussed under pitch hardly support the definition of pitch as an attribute of a sensation: rather they are essentially physical phenomena, such as Doppler effect, interference of high frequency sounds, Aeolian tones, jet tones and the frequency ratios of musical scales. Often (unfortunately) throughout the book the word pitch appears as a substitute for the frequency of a physical vibration.

The international standard for the frequency of the tuning *A* is said to be 439 cycles per second (it never was an international standard and not the British standard since 1939!), although in Chapter XI the *A* is given the correct frequency of 440 cycles per second. The myth of temperature dependence is perpetuated by "Since the pitch of a note generally alters with temperature it is of course necessary to define a temperature dependence when adopting a standard." It is evident that the very complete story in British Standard 880: 1950 did not make its way to the author.

Despite its shortcomings, this book is worthy of consideration in those elementary physics courses which extend over several semesters and which devote several months to acoustics. It should prove useful for elementary reference work.

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